# Bond Portfolio Creation Strategies 

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## I. Assumptions

Given the parameters of this situation, certain assumptions had to be made in order to proceed with the analysis. The $5 \%$ discount rate is assumed to be an annual interest rate that is compounded quarterly, which translates into a $1.25 \%$ quarterly interest rate. Additionally, it is assumed that all the liabilities are due at the end of each quarter, meaning that the first liability is discounted when determining its present value. In order to simplify calculations, accrued interest is ignored and surplus coupon payments between quarters do not accrue interest. The purpose of this investment report is solely to minimize the amount of money spent to pay off the listed obligations. As such, only bonds that mature within 3 years are used. In terms of the cash flow matching method for portfolio formulation, a key assumption that is made is that the cash flows are matched using coupon payments and face value payments from the bonds, as opposed to the net present value of the bonds at the time the liabilities are due. This means that to cover the liabilities, the coupon payments are sufficient, and bonds don't need to be sold. The immunization method does not require coupon payments to fully cover liabilities; instead, bonds may be sold to cover the obligations.

## II. Modeling

Modeling for this investment report is done via Microsoft Excel with major utilization of the built-in Excel Solver application. In order to determine the yield to maturity, the standard bond price formula is used. This formula, in conjunction with the Excel Solver application finds the yield to maturity of each bond. This, in turn allows the selling price of the bond to found, with the aid of the bid/ask spread. Finally, using the Macaulay Duration as a subtotal, the modified duration can be determined for each bond.

To find the bonds to be used in the portfolio used to pay the given liabilities, a simple optimization model is used. The overall goal is to minimize the initial investment of cash, which is a function of the amount of each bond that is purchased. The constraints of the optimization model are two-fold. First, the amount of each bond purchased has an upper bound and lower bound, the amount of the bond that is available at the time the purchase is made and $\$ 0$, respectively. Second, the coupon payments in a given quarter plus the surplus from previous quarters, less the end of quarter obligation, must be greater than $\$ 0$, for every quarter. In other words, the coupon payments for the bonds in every quarter plus any surplus must be enough to cover the liabilities for each quarter.

The model for the immunization model is largely the same, with the same overall goal, a minimization of the initial investment. The constraint on liability coverage for individual quarters is removed, and constraints on present values and duration are added. More specifically, the present value of the portfolio must equal the present value of the obligations. Additionally, to satisfy the second immunization equation, the duration of the portfolio must correspond to the duration of the obligation as well.

## III. Cash Flow Matching

## A. Portfolio Structure

| 00138PAA | $\$ 75,918.22$ |
| :--- | :--- |
| 001392AA | $\$ 757,190.00$ |
| 00163XAC | $\$ 157,800.00$ |
| 001957AS | $\$ 307,740.00$ |
| 00206HH7 | $\$ 306,660.00$ |
| 002824AG | $\$ 160,530.00$ |
| 1 Month | $\$ 19,965.99$ |
| 2 Year | $\$ 2,917,153.05$ |
| Total: | $\$ 4,702,957.26$ |

The portfolio structure shown on the left was created using the cash flow matching method. The total investment required for this Portfolio is approximately $\$ 4.7$ million with duration of 2.035 years. With interest rate changes of 100 basis points in either direction, the NPV of the portfolio only changes about $2.1 \%$ and the NPV of the liabilities changes $2.7 \%$. However, as explained later in the advantages of cash flow matching, interest rate changes don't impact this portfolio negatively.

## B. Advantages

The method of cash flow matching produces coupon payments and bond face value repayments at the exact times necessary to pay off all obligations. Under this method, the payment in full of all liabilities is guaranteed (absent bond defaults) because the coupon payments are at a fixed rate, which means cash flows are predictable. Under this method, when a liability is due, there will be no problem paying off the debt, regardless of interest rates.

Changes in interest rates change the prices and yields of bonds, but under the cash flow matching method of portfolio construction, this is irrelevant in paying off debts. The guaranteed effectiveness of cash flow matching makes it a very attractive method for portfolio construction.

## C. Disadvantages

Unfortunately, the cash flow matching methodology has many shortcomings. First, the portfolio created via cash flow matching costs more than the portfolio created via immunization. The reason for this is that the bonds selected must give coupon payments at the correct quarters and in the correct amounts, to meet the obligations as they come due. This eliminates certain combinations of cheaper bonds that do not generate enough income to pay liabilities in a given quarter, but that might yield more in the long run. Second, since the overarching goal is to minimize the investment and just to pay obligations due within 3 years, only cash flows within 3 years are counted. For this reason, the best bonds, according to this method, are bonds that mature within 3 years (because they return their face values within the 3 years). Unfortunately cash flow matching is very restrictive, and since the scope of this project doesn't include reinvestments of surplus, a lot of surplus payments are carried over between quarters and aren't used to generate more capital. For this reason, the NPV of the portfolio is $\$ 4.5$ million.

To conclude the discussion on cash flow matching, it is important to note that the appeal of the approach is the guarantee of being able to pay off liabilities in full and on time. The price for this guarantee is a premium in the portfolio cost because one must be selective when creating a portfolio using this method, since precise coupon payments must be made at specific times.

## IV. Immunization

## A. Portfolio Structure

| 00104PAC | $\$ 10,140.54$ |
| :--- | :--- |
| 00138QAA | $\$ 11,221.31$ |
| 00139PAA | $\$ 66,849.27$ |
| 001814AR | $\$ 26,441.17$ |
| 001920AB | $\$ 12,554.30$ |
| 00209TAA | $\$ 15,665.40$ |
| 00209TAB | $\$ 36,435.73$ |
| 2 Year | $\$ 3,971,573.63$ |
| 7 Year | $\$ 6,919.30$ |
| 10 Year | $\$ 27,179.95$ |
| 20 Year | $\$ 73,416.69$ |
| 30 Year | $\$ 108,391.36$ |
| Total | $\$ 4,366,788.65$ |

The portfolio structure shown on the left was created using the immunization method. The total investment required for this Portfolio is approximately $\$ 4.35$ million with duration of 2.81 years which is equivalent to the duration of the liabilities (a constraint of the optimization equation used here). With interest rate changes of 100 basis points in either direction, the NPV of the portfolio changes approximately $2.6 \%$ and the NPV of the liabilities changes $2.7 \%$. There is a very close correlation between interest rate changes and the NPV of the liabilities and the portfolio because the durations for both are the same, as well as their present values (second constraint of optimization equation.)

## B. Advantages

Using Immunization allows the creation of a portfolio that gets nearly maximum yield, and at the same time, allows for the payment of liabilities. This method yields a portfolio that costs approximately $7.1 \%$ less than the porffolio created via cash flow matching. Immunization is much less restrictive than cash flow matching because the coupon payments aren't the only income used to pay liabilities. Coupon payments as well as bond sales contribute to the capital used to pay off obligations. Immunization doesn't require bonds with durations smaller than 3 years, in this scenario. In fact, since the duration of the liabilities given is approximately 2.81 years and all the bonds that mature within 3 years, do not have durations as high as 2.81 , the immunization method actually requires bonds that have longer durations than the bonds that mature within 3 years. This is an advantage over cash flow matching because if interest rates decrease, the bonds in the portfolio can be sold for a higher price than they were bought for. Under the cash flow matching system, no income is generated from bonds after 3 years, because all the bonds chosen for that portfolio mature within 3 years.

## C. Disadvantages

Immunization has its fair share of disadvantages as well. A portfolio created via Immunization does not guarantee that capital is available to pay off liabilities at every quarter. In general, the portfolio value will be such that selling bonds will allow a fulfillment of the obligations, however, it is not guaranteed since coupon payments will often not be enough to pay off the liabilities. Radical changes in interest rate can cause the portfolio NPV to fall below the NPV of the liabilities, which means that even after selling the bonds; (at a reduced price) obligations might not be met. In addition to this uncertainty, there is also a possibility that a liability is due at a time where interest rates are high, which means that the bonds which need to be sold to cover the obligation, would have to be sold at a great loss, since prices on those bonds would be down. Another problem with immunization is that interest rate changes require, or at least make it a prudent idea to re-immunize the portfolio. These uncertainties increase the risk of using an immunized portfolio, but with that risk, comes the reward of a cheaper (than cash flow matching) portfolio.

## V. Integrated Approach

A. The Approach and Portfolio Structure

| 00138PAA | $\$ 100,457.05$ |
| :--- | ---: |
| 001392AA | $\$ 757,190.00$ |
| 001957AS | $\$ 307,740.00$ |
| 00206HH7 | $\$ 306,660.00$ |
| 002824AG | $\$ 160,530.00$ |
| $\mathbf{1}$ Month | $\$ 19,965.99$ |
| $\mathbf{2}$ Year | $\$ 2,458,410.76$ |
| 30 Year | $\$ 167,418.83$ |
| Total | $\$ 4,278,372.63$ |

An integrated approach using a combination of cash flow matching and immunization is possible. The methodology is to first cash use cash flow matching to meet a subset of the obligations. The rest of the obligations can be immunized. In this approach, since the final liability due is disproportionately large when compared to the other liabilities, this liability is immunized while all the rest of the liabilities are paid using cash flow matching.

## B. Advantages

The advantages of this approach are similar to the advantages of the immunization and cash flow matching approach by themselves. First, as with cash flow matching this portfolio guarantees the payment of all liabilities (except the last one) via face value and coupon payments. Secondly, the portfolio is immunized against the last liability to allow bonds to be sold at the end of the period to pay off the final liability. The overall cost of this portfolio is $9.1 \%$ lower than full cash flow matching and $2.2 \%$ lower than full immunization.

## C. Disadvantages

Unfortunately, due to the large value of the final liability, its repayment is not guaranteed because it is not cash flow matched. This is the major drawback of this mixed approach. Though this approach is much less expensive than both the other approaches to portfolio creation, the lack of a guaranteed solvency for the last liability is what allows this portfolio to cost less than the other options.

## VI. Conclusion

Ultimately, portfolio selection is based on risks and rewards. Each type of Portfolio creation has it's own merits and disadvantages. Cash flow matching alone produces mid-range yields, but guarantees solvency of all liabilities on time and in full. Immunization has much more uncertainty and is affected by interest rate changes, and large changes mean that obligations might not be able to be paid off. Additionally, immunization allows bonds to be sold, which could be for profit, or for a loss. However, this uncertainty may pay off because these portfolios in the long run have the largest yield and are quite cheap to purchase. Finally, the combination portfolio guarantees payment for a subset of liabilities but not for the final liability. The combination portfolio also does not yield as much as the immunization portfolio, but does cost less than both the cash flow matching portfolio and the immunized portfolio.

In the end, the portfolio that should be chosen is based directly upon what the ultimate goal is. In this scenario, the payment of liabilities and minimization of initial investment is paramount, which would make cash flow matching or the combination portfolio. If yield is a major factor, then immunization should be chosen. Finally, if a combination of yield and solvency guarantees are needed, then the integrated approach is preferred.

