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Investment Science

Project #2 – A Fund of Funds

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I. Introduction and Definitions

In order to find a combination of mutual funds (Diversified Growth, Diversified Income, and Short-Term Bonds) that closely mimic each of the given individual allocations (A, B, C and D), one must understand the concepts of risk, reward, expected and actual return, variance, and covariance. Investment science is based on the principles of risk and reward. Investors are rewarded for taking on varying levels of risk. The concept of reward is embodied in the terms actual and expected return while the concept of risk is embodied in the terms variance and covariance.

The expected return of an asset is the amount with respect to the initial investment that an investor expects to receive for investing in the asset. The key term in the phrase is *expected*, since the return of assets is often not fixed and must be estimated. Alternatively, the expected return can be thought of as the amount an investor would receive per investment unit if they invested in the asset an infinite amount of times. For example, if an asset has an expected return of 10% and the initial investment is $1000, after a given period of time the investor would expect to receive $1100 in return for their initial investment (initial investment of $1000 + the 10% return of $100). The *actual* return from the asset may or may not match the expected return.

Variance and covariance both serve as an indicator of how different actual rate of returns are likely to be from expected rates of return. Specifically, if an asset has a high variance, it is more volatile and one can expect that the actual rate of return has a propensity to vary from the expected rate of return. Conversely, an asset with a low variance will often yield dividends relatively close to the expected returns. When calculating the variance of a portfolio comprised of multiple assets, one must take into account the mutual dependence between assets. This mutual dependence can be summarized by a concept called covariance. A large value (absolute value) for the covariance means that two assets are strongly correlated while a small value means that the assets are weakly correlated. A positive covariance means that an increase in one asset usually implies an increase in the other asset and a negative covariance means that the assets are inversely correlated.

This report uses the concepts of reward and risk to create a portfolio consisting of mutual funds that in turn consist of individual assets. This portfolio's goal is to match the behavior of 4 given sets of investment allocations. The resulting portfolios generate a return and variance equivalent to the given investment allocation's return and variance. Assets with the same expected return and variance usually behave almost exactly alike because their expected returns and the propensity for those returns to vary are also the same. Finally, alternative portfolios will be presented that have the same variance as the given allocations, but yield a greater return.

II. Methodology and Procedural Overview

Note: Mathematical details will not discussed in detail in the body of this report, as the objective of this report is to highlight criteria for closeness of fit and not to elaborate on the mathematical details of variance and covariance. Technical details used in this section are included in the technical appendix.

In order to analyze the given allocations, the expected return and the variance of each of the individual allocations needs to be determined. The expected return can be calculated using the weights of each asset in the portfolio and the expected return for each of those assets. The

	A	B	C	D
LC	16%	25%	38%	50%
SC	5%	8%	12%	16%
IS	4%	7%	10%	14%
IG	28%	28%	20%	13%
HY	7%	7%	6%	4%
IB	5%	5%	4%	3%
STB	35%	20%	10%	0%
Return	6.858%	7.675%	8.585%	9.485%
Variance	0.261%	0.469%	0.854%	1.371%

Table 1

variance of each allocation is calculated by first calculating the covariance between each of the seven asset classes. Then, using the resulting covariance values and the weights of each individual asset class in the allocation as a whole, the variance of the allocation can be calculated. **Table 1** at the left shows each of the component asset classes, the expected return, and variance of each of the four given allocations (A-D). An analogous procedure is used to calculate the expected return and variance for each of the individual mutual funds, shown in **Table 2**. The third step in creating the mimicking portfolios is to find the covariance between each of the three different portfolios. Once the matrix to calculate the variance of varying weights of each mutual fund is created, a simple optimization algorithm finds mimicking portfolios.

The optimization equation seeks to create a portfolio consisting of three separate weights, one for each mutual fund. The constraints on the optimization equation are four-fold. First, the total weights of each of the three funds must sum to one. Second, each weight must be positive (shorting portfolio assets is beyond the scope of this report). Finally, both the variance and return must be equal to the variance and return of the allocation that is to be mimicked.

	Diversified Growth	Diversified Income	Short-Term Bonds
LC	50%	15%	0%
SC	27%	0%	0%
IS	23%	0%	0%
IG	0%	39%	0%
HY	0%	23%	0%
IB	0%	20%	0%
STB	0%	3%	100%
Return	10.270%	7.103%	4.750%
Var.	1.966%	0.256%	0.063%

Table 2

III. Results of Fitness Calculations

	Diversified Growth	Diversified Income	Short-Term Bonds
Allocation A	14.855%	54.728%	30.416%
Allocation B	32.023%	49.196%	18.781%
Allocation C	55.701%	32.320%	11.980%
Allocation D	78.417%	17.275%	4.308%

Table 3

The solutions to the optimization equations highlighted in section II of this report yield the weights of each mutual fund needed to mimic the given allocations (**Table 3**). For example, in order to obtain a return of 6.858% and variance of 0.261% (**Table 1**) one must structure a portfolio to have 14.855% of the capital invested in the diversified growth mutual fund, 54.728% of the capital invested in the diversified income mutual fund, and 30.416% of the capital invested in the short-term bonds mutual fund (**Table 3**). In general, one can notice the trend that the aggregate weights of each of the individual asset classes within each “fund of funds” is quite close to the original allocation percentages (**Table 1**). It is important to note that all weights for each individual allocation sum to approximately 1, the weight for the whole portfolio.

IV. Criteria for Fitness Calculations

The ultimate objective of this report is to create a portfolio consisting of mutual funds that most closely mimics the behavior of the original allocations given. The characteristics defining the behavior of an allocation are the expected return of the allocation and the probability that the actual return will be relatively close to the expected return (variance is used to deduce this). Both these characteristics can be recreated in a portfolio of different assets, as long as the expected return and variance match the values of the original allocation. In addition to being intuitively evidenced, this criterion for closeness of fit is also mathematically sound. In many fields of probability, the mean (expected rate of return, in this case) and standard deviation (square root of variance) are enough to characterize a distribution and two distributions with the same mean and standard deviation generally behave in the same fashion.

V. Alternative Criterion

	Diversified Growth	Diversified Income	Expected Return	Original Exp. Return
Allocation A	0.872%	99.128%	7.130%	6.858%
Allocation B	25.097%	74.903%	7.897%	7.675%
Allocation C	51.761%	48.239%	8.742%	8.585%
Allocation D	77.083%	22.917%	9.544%	9.485%

Table 4

The alternative criterion selected is one that keeps the variance of the original allocation, but seeks to maximize the return. In other words, this approach seeks to maximize the return for the given risk. The volatility in this portfolio is equivalent to the original allocations given in section III, but the return is greater since the optimization equation used seeks to maximize return, instead of constraining it to equal the return of the original allocation. The advantage of this criterion is that the portfolio incurs the same risk as the close-fitted portfolio, but the reward is greater.

With the increase in reward, even though the variance percentage remains the same, the overall variance increases. Two different portfolios with a variance of 5% might have different absolute variances if one portfolio has a larger return than the other. The closeness of fit, or propensity for the portfolios created by this method to mimic the original allocations is still relatively high, but less than that of the original criteria introduced in this report.

Finally, it is important to note that none of the fund of funds created by this method include the short-term bond mutual fund. This is because the short-term bond mutual fund is normally used to decrease the variance of a portfolio; however, it lowers the overall return of the portfolio. A sufficiently low variance can be achieved without the use of the short-term bond mutual fund, which means that its low rate of return precludes its usage in this method.

VI. Conclusion

The two criteria proposed in this report allow the investor to choose a fund of funds that either mimics their original investment allocation exactly or one that mimics the volatility of the original investment allocation, but maximizes the return. These are two possible criteria used to create a portfolio consisting of mutual funds that consist of assets in seven different asset classes. These methods show that using the expected rate of return and volatility of a portfolio, the behavior of less diversified portfolios can be mimicked in a more diversified portfolio.

Technical Appendix

Step 1: Create a matrix of the correlation coefficients given.

	Correlation Coefficients						
	LC	**SC**	**IS**	**IG**	**HY**	**IB**	**STB**
LC	1	0.7	0.6	0.4	0.5	0.1	0.3
SC	0.7	1	0.4	0.2	0.5	0	0.1
IS	0.6	0.4	1	0.2	0.3	0.4	0.1
IG	0.4	0.2	0.2	1	0.3	0.3	0.9
HY	0.5	0.5	0.3	0.3	1	0	0.2
IB	0.1	0	0.4	0.3	0	1	0.3
STB	0.3	0.1	0.1	0.9	0.2	0.3	1

Step 2: Calculate the covariance between the asset classes. Each element of the matrix is calculated by finding the product of the x-axis volatility, y-axis volatility, and the corresponding (x,y) value in the matrix created in step 1.

		Covariance Between Asset Classes						
	volat.	0.15	0.18	0.17	0.05	0.06	0.09	0.025
volat.		**LC**	**SC**	**IS**	**IG**	**HY**	**IB**	**STB**
0.15	**LC**	0.0225	0.0189	0.0153	0.003	0.0045	0.0014	0.0011
0.18	**SC**	0.0189	0.0324	0.0122	0.0018	0.0054	0	0.0005
0.17	**IS**	0.0153	0.0122	0.0289	0.0017	0.0031	0.0061	0.0004
0.05	**IG**	0.003	0.0018	0.0017	0.0025	0.0009	0.0014	0.0011
0.06	**HY**	0.0045	0.0054	0.0031	0.0009	0.0036	0	0.0003
0.09	**IB**	0.0014	0	0.0061	0.0014	0	0.0081	0.0007
0.025	**STB**	0.0011	0.0005	0.0004	0.0011	0.0003	0.0007	0.0006

Step 3: The next step is to calculate the variance and expected return of each individual allocation. Each element of the matrix is calculated by finding the product of weight squared and the corresponding (x,y) value in the matrix created in step 2. The expected return is a weighted average of the expected returns of each individual asset class. The variance of the portfolio is the sum of the grid values.

	Weights	0.16	0.05	0.04	0.28	0.07	0.05	0.35
Weights		**LC**	**SC**	**IS**	**IG**	**HY**	**IB**	**STB**
0.16	**LC**	0.00057600	0.00015120	0.00009792	0.00013440	0.00005040	0.00001080	0.00006300
0.05	**SC**	0.00015120	0.00008100	0.00002448	0.00002520	0.00001890	0.00000000	0.00000788
0.04	**IS**	0.00009792	0.00002448	0.00004624	0.00001904	0.00000857	0.00001224	0.00000595
0.28	**IG**	0.00013440	0.00002520	0.00001904	0.00019600	0.00001764	0.00001890	0.00011025
0.07	**HY**	0.00005040	0.00001890	0.00000857	0.00001764	0.00001764	0.00000000	0.00000735
0.05	**IB**	0.00001080	0.00000000	0.00001224	0.00001890	0.00000000	0.00002025	0.00001181
0.35	**STB**	0.00006300	0.00000788	0.00000595	0.00011025	0.00000735	0.00001181	0.00007656
Variance	0.00260554							

Technical Appendix

Step 4: Using the same method in step 3, calculate the expected returns and variance for each of the individual mutual funds. This same table can be used to calculate the covariance between the 3 mutual funds.

Step 5: Using the following table and constraints, find solutions for x,y, and z using Excel's built-in solver. Note: Excel pseudo-functions are provided to show the contents of each grid element.

	weights	X	Y	Z
weights		**Fund 1**	**Fund 2**	**Fund 3**
X	**Fund 1**	X*X*var(1)	X*Y*cov(1,2)	X*Z*cov(1,3)
Y	**Fund 2**	X*Y*cov(1,2)	Y*Y*var(2)	X*Z*cov(2,3)
Z	**Fund 3**	X*Z*cov(1,3)	X*Z*cov(2,3)	Z*Z*var(3)

Constraints:

1) X+Y+Z = 1.0
2) variance(sum of grid) = variance of original allocation
3) E(X) * w(X) + E (Y) * w(Y) + E(Z) * w(Z) = E(original alloc.)
   Where E(X) is the expected return of X.
4) Maximize Return, instead of constraint #3 if using alternative criteria.