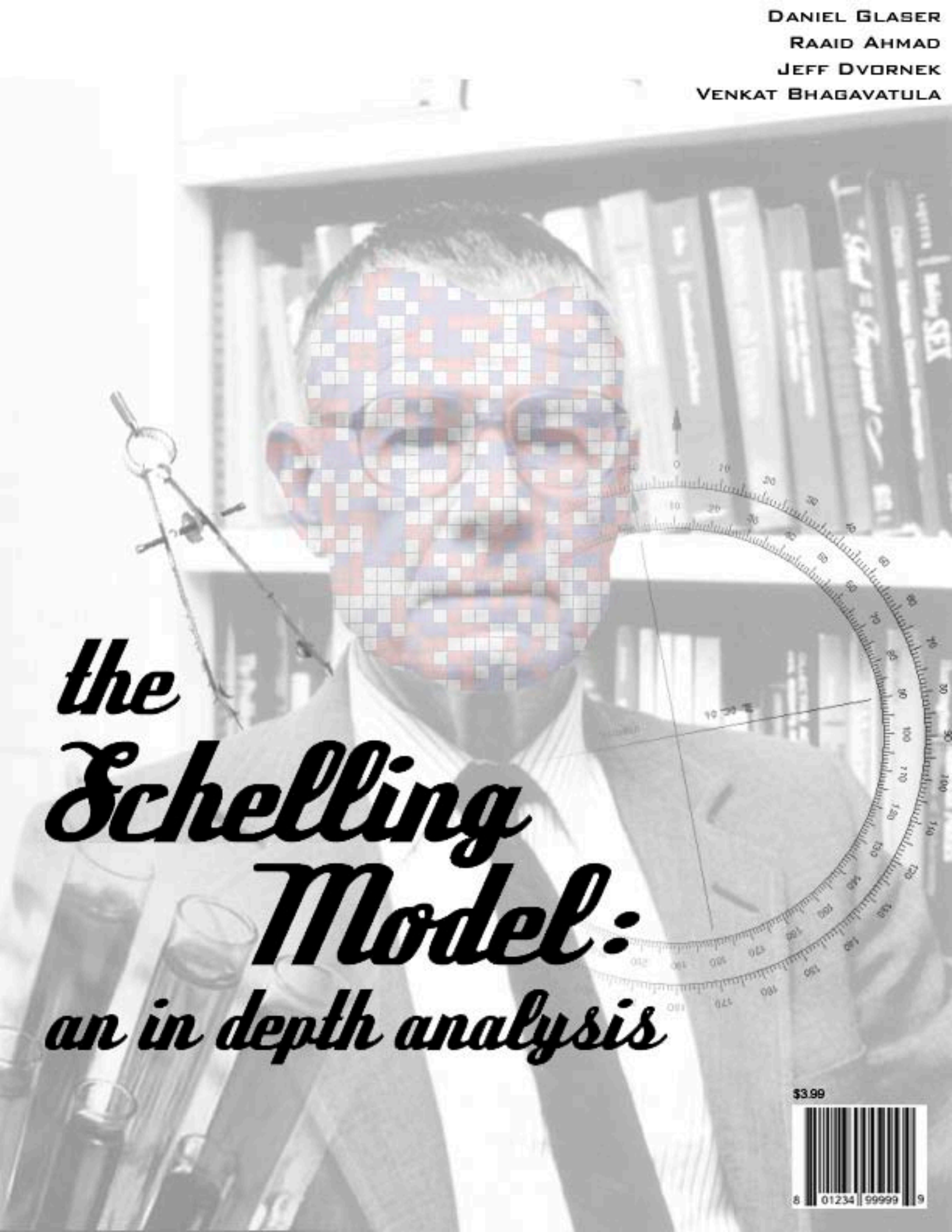


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*the  
Schelling  
Model:  
an in depth analysis*

\$3.99



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## I. Schelling Model Overview

### A. Purpose –

The purpose of the Schelling Model is to determine segregation levels of populations given certain characteristics of the population size and attributes of individuals. More generally, the Schelling Model seeks to understand how attributes of individuals in a population affect the segregation and migration patterns of the society as a whole.

### B. The Key Attribute: Happiness –

The key attribute for an agent is its preference level  $p$ . The preference level  $p$  is standard to almost all Schelling Models and it denotes the level of “happiness” an individual requires in order for it to be considered happy in terms of the simulation. The happiness of an individual agent is a measure of the percentage of agents adjacent to it that are of the same color. The higher the number of adjacent agents that are the same color, the higher the happiness for that agent will be. The happiness ratio for an agent is determined by the following formulae:

For Red Agents:  $[\text{Number of Reds}] / [(\text{Number of Blues}) + (\text{Number of Reds})]$

For Blue Agents:  $[\text{Number of Blue}] / [(\text{Number of Blues}) + (\text{Number of Reds})]$

The important point to remember is that the denominator is not necessarily 8, even though at any given time, an agent is adjacent to 8 other squares. If an agent is isolated and has no neighbors, it has a happiness ratio of 1.0, meaning that it is content with its position. As such, the critical points that necessitate an extra neighbor of the same color for happiness are not necessarily .125, .25, .375, etc. **Figure 1** denotes critical points for various neighbor combinations.

Total Neighbors	0 - 1	1 - 2	2 - 3	3 - 4	4 - 5	5 - 6	6 - 7	7 - 8
8	0.125	0.250	0.375	0.500	0.625	0.750	0.875	1.000
7	0.143	0.286	0.429	0.571	0.714	0.857	1.000	
6	0.167	0.333	0.500	0.667	0.833	1.000		
5	0.200	0.400	0.600	0.800	1.000			
4	0.250	0.500	0.750	1.000				
3	0.333	0.667	1.000					
2	0.500	1.000						
1	1.000							

**Figure 1**

## II. Segregation Measurement Metric

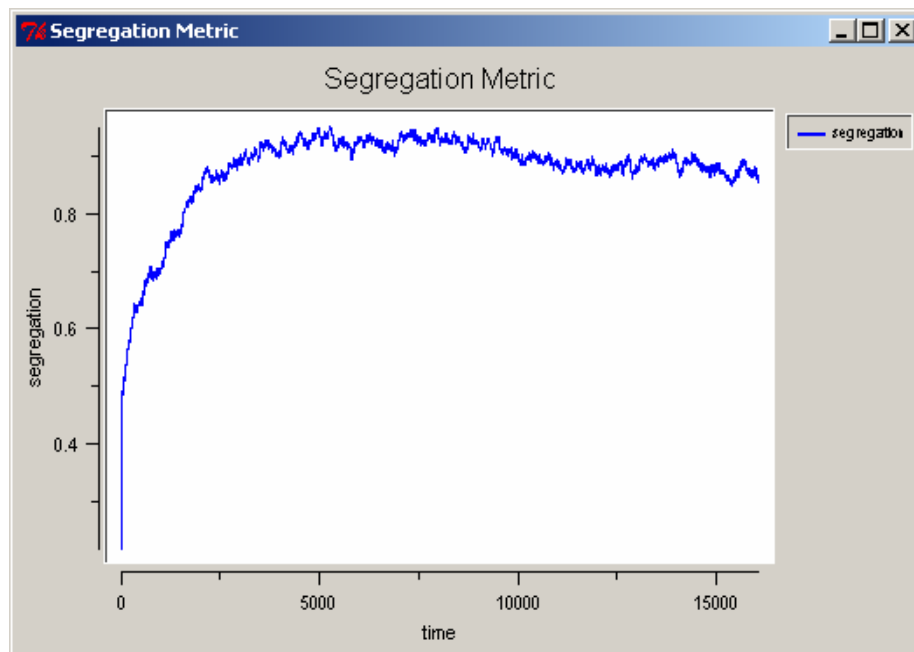
To measure the degree of segregation present in the Schelling world, we used an implementation of the Dissimilarity Index proposed by Duncan and Duncan<sup>1</sup>. The index is calculated with respect to a partition of the world. In each of these parts, or neighborhoods, the relative segregation is determined by finding the difference in the percentage of each type of agent's total population that lies in that neighborhood. The amount of each of the two populations  $R$  and  $B$  in each neighborhood, respectively, are denoted as  $r_i$  and  $b_i$ . The difference in percentage is summed over all  $i$  neighborhoods to give

$$D = 0.5 \sum_{i=1}^n \left| \frac{r_i}{R} - \frac{b_i}{B} \right|$$

**Figure 2**

The Dissimilarity Index,  $D$ , takes values between zero and one, zero being representative of equivalent distributions of the two populations across the set of neighborhoods, one being representative of complete isolation of the populations. Values of  $D$  that are below 0.3 are considered indicative of low segregation whereas values above 0.6 are considered indicative of high segregation.

The metric was measured by graphing  $D$  vs. time and examining the value after the simulation halts. An example chart of the metric:



**Figure 3**

<sup>1</sup> Duncan, O. and Duncan, B. (1955a) "A methodological analysis of segregation indexes", *American Sociological Review*, 20, 210-217

### III. The Initial Schelling Model Analysis

#### A. *Behavior and Characteristics*

For each agent present in the world, a cell that would increase the agent's current happiness will be chosen at random during each step. If such a location cannot be found, the agent will remain in its current cell. If an improvement can be made, the agent will swap with whatever occupies this new "target cell," provided the cell is not currently occupied. This continues until no movements occur in each subsequent step.

#### B. *Experimentation and Graph*

Using the selected segregation metric, we ran 63 trials on the original Schelling model provided to us: 1 for 1200, 800, and 400 agents of each type at each happiness threshold level from 0 to 1 at increments of 0.05.

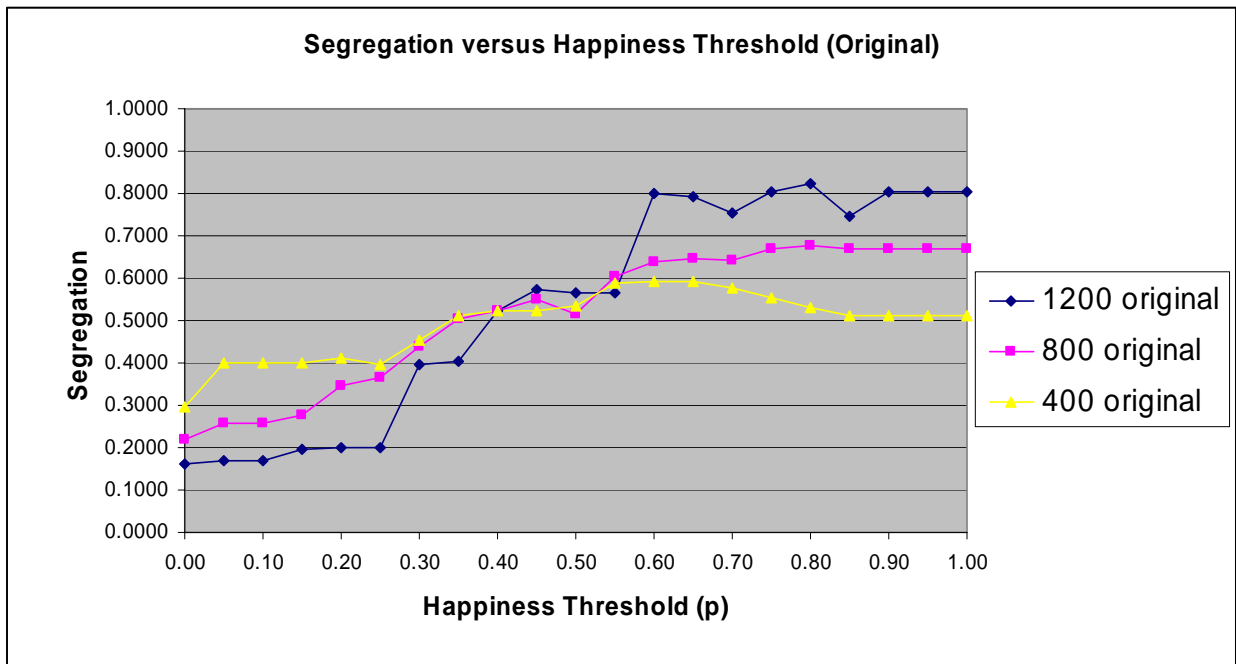


Figure 4

#### C. *Analysis and Summary*

We discovered pronounced differences in the progression of the three denominations. The trials involving 400 agents of each type interestingly started with higher segregation (on lower thresholds of happiness) than the other two groups. As two groups of 400 yield a relatively low population density, we determined that their higher level of segregation was due to the fact that small isolated groups would be easier to form, which contributes greatly to the level of segregation. This phenomenon also explains why two groups of 1200 start at such a low segregation, as a population with a higher density is not as naturally inclined to be segregated.

It is important to notice that in the progression of all three groups, as the happiness threshold increases, there is a *rapid* increase in segregation at 0.25. This can be explained by the fact that since each agent bases its happiness on potential contact with 8 other agents, there will be threshold increases that demand an agent to seek contact with one more agent of the same color than was previously necessary to fulfill its happiness requirements. The fact that 0.25 is equal to  $2/8$  is significant, since this means that at this threshold, every agent must now seek out three agents of its own color rather than two. This would certainly increase segregation, as agents will be more inclined to seek out other agents of the same color.

As the threshold increases to 0.5, agents must balance between seeking out other similar agents and shying away from those different. This explains the convergence of the three population densities.

When the threshold requires that a majority of an agent's neighborhood be of like type, we find that the higher population densities exhibit far greater segregation. In fact, under these conditions, agents in a lower population density tend to decrease in segregation. This is due to the fact that since there are fewer agents, it is less likely that an agent can find a place to move with a suitable ratio of neighbors.

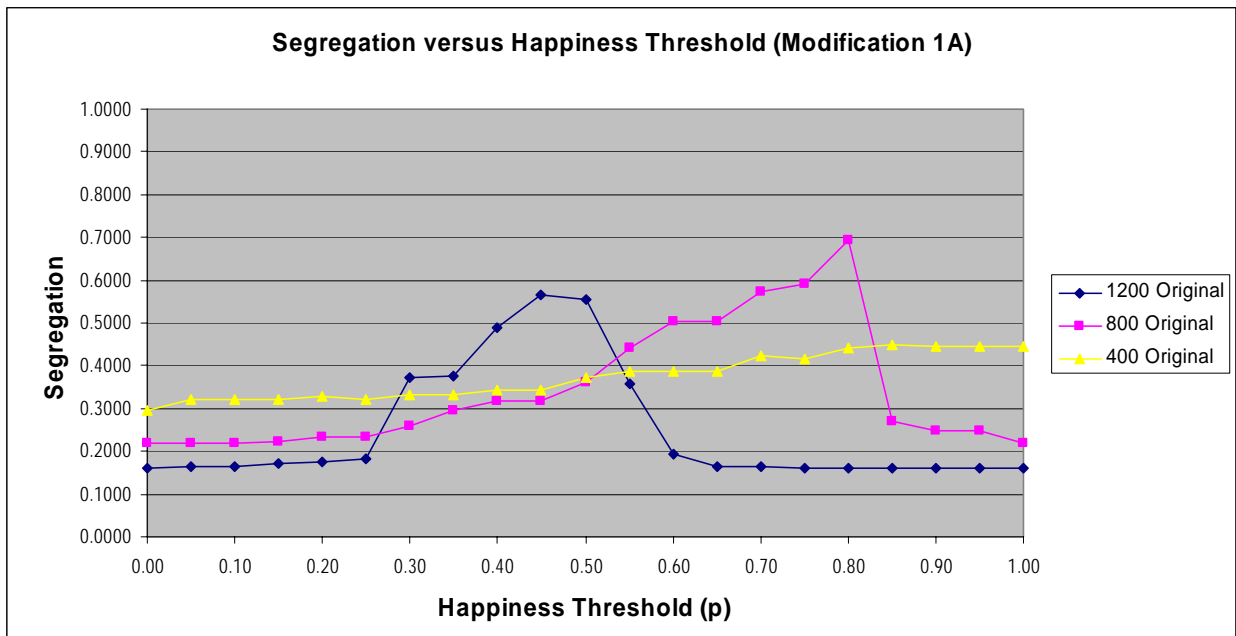
#### IV. Modification 1A (Distance Bias):

##### A. Behavior and Characteristics

This modification changes the movement behavior of the agents so that each agent moves to closer cells as opposed to cells that are further away. The rules for agent movements are as follows:

- If the agent's happiness is  $\geq p$ , the agent will not move.
- If the agent's happiness is  $< p$ , the agent moves to the closest cell such that its happiness will be  $> p$  after moving to the new cell.
- If no cell fulfills the agent's happiness threshold, it will not move.

##### B. Experimentation and Graph



**Figure 5**

Our tests were performed on a 50x50 world with 1200, 800, and 400 agents of each color. **Figure 6** illustrates the percentage of the world that is occupied under each of the 3 test conditions.

Agents per Color	% of World Occupied
400	32%
800	64%
1200	96%

**Figure 6**



### C. Distance Formula

The distance calculation function used in modification 1A and 1B is the standard distance formula applied 4 times. The reason the distance formula needs to be applied 4 times is because of the wraparound nature of the world. The distance calculations are:

$$\sqrt{(X_f - X_i)^2 + (Y_f - Y_i)^2}$$

$$\sqrt{[(\text{worldXSize}) - (X_f - X_i)]^2 + (Y_f - Y_i)^2}$$

$$\sqrt{(X_f - X_i)^2 + [(\text{worldYSize}) - (Y_f - Y_i)]^2}$$

$$\sqrt{[(\text{worldXSize}) - (X_f - X_i)]^2 - [(\text{worldYSize}) - (Y_f - Y_i)]^2}$$

Once each of these formulae is calculated, the shortest distance is chosen, because that is presumably the path the agent would take to reach the destination cell. These four calculations account for normal movement and wraparounds.

### D. 2400 Agents – High Density

In this test, 96% of the world is occupied, which means that almost all of the agents have 8 neighbors and only under rare occurrences will an agent have 6 neighbors. The expected value of the number of initial neighbors per agent is 7.68. Under these circumstances, the critical points at which extra agents will be needed for a change in happiness are largely based around .125, .25, .375, .5, .625, .75, .875, and 1.0 (Assuming 8 neighbors).

For values of  $p < .25$ , the segregation index has a slow rate of change and only increases from .16 to .18 on the interval  $0 \leq p \leq .25$ . This indicates that if agents in the simulation have a preference for 0,1 or 2 neighbors of the same color, the final segregation index of the population will be considered low. On the interval  $.25 < p \leq .375$ , the segregation index jumps rapidly, levels out, and then jumps again at the end of the interval. The likely reason for the initial jump at .25 is because this is the point at which an agent needs to be adjacent to 3 agents of the same color in order to be happy. The segregation index is level on the interval  $.3 < p \leq .35$  because there is no change in the number of agents of the same color that must be adjacent to another agent to make it happy. On the interval  $.375 < p \leq .5$ , the segregation index hits a peak and levels off. The reason this is a critical point is because for  $p > .5$ , an agent must be adjacent to 4 or more agents of like type in order to be happy.

On the interval  $.375 < p \leq .5$ , this modification starts to have a profound effect on the final segregation index. Every agent moves to the closest space that satisfies its happiness ratio. Once an agent moves, the space it has moved from becomes empty, however, the reason the agent moved from the space is because the happiness ratio at that space was *lower* than the agent's happiness threshold. This means that no agent of the same color will move to that cell until the happiness ratio of that cell increases. For sufficiently large values of  $p$ ,

this restricts the movement of agents greatly because agents moving at the end of any given step have very few spaces to move to. This, combined with the attribute of agents to not move if they are happy, makes it very difficult for spaces with a high happiness ratio to open up.

On the interval  $.5 < p \leq .625$ , the segregation index plummets because instead of needing 3/7 or 4/8 neighbors to be of the same color to achieve happiness, an agent would require 4/7 or 5/8 neighbors to be of the same color to achieve happiness. This requirement prevents an agent from latching onto the outskirts of a group because its happiness would still not be met.

On the interval  $p > .625$ , the curve reaches a final segregation index of .16. The reason for this is that the high value of  $p$  causes agents to freeze up rather quickly, after a short number of turns, leaving many agents without any options for movement. An agent is frozen and ceases movement permanently if its happiness ratio is met or if there are not empty cells that fulfill its happiness ratio.

#### E. 1600 Agents – Medium Density

In this test, 64% of the world is occupied, which means that on average, an agent has 5.119 neighbors. Under these circumstances, the critical points at which extra agents will be needed for happiness are largely based around .2, .4, .6, .8, and 1.0 (Assuming 5 neighbors).

A large discrepancy between the segregation index in a field of 2400 agents versus a field of 1600 agents is on the interval  $.25 < p \leq .5$ . The rate of increase of the segregation index in a field of 1600 agents is significantly slower because more agents have empty space around them and the agents that are in a small bundle with no agents of differing color have a happiness ratio of 1.0 and have no incentive to move. Additionally, since some agents are isolated, and have no neighbors, their happiness ratio is 1.0, causing them to remain somewhat isolated in areas surrounded, but not immediately surrounded, by different colored agents.

This curve represents a steady growth of the segregation index until the peak at  $p = .8$ , at which point the segregation index declines rapidly for higher values of  $p$ . At first glance, it may not be readily apparent why the segregation index of this population peaks at  $p = .8$ , while the peak segregation of a 2400 agent world is  $p = .45$ . However, after careful examination, it can be seen that since the average agent has 5 neighbors, at  $p = .8$ , an average agent needs *all* of its neighbors to be of the same color. This is one of the main reasons the segregation index spikes between .8 and .85. The low rate of change of the curve is explained above in terms of frozen agents given sufficiently high values of  $p$ .

#### F. 800 Agents – Low Density

This curve increases very gradually and the segregation index on the interval  $.0 \leq p \leq .95$ , nearly the entire spectrum of values for  $p$ , only rises from .3 to .45. The main reason for this is the fact that on average, an agent only has 2.56 neighbors. This means that many neighbors are isolated and do not move at all. Agents will pick the closest cell that meets their happiness ratio and with many (68%) of the cells open for occupancy many agents will be happy after 0 or 1 moves, meaning that segregation never has a chance to develop. Even with relatively high values of  $p$ , the massive amounts of open space allows free movement of

agents and only leads to a small increase in segregation since agents search for small, bunched groups.

### *G. Analysis and Summary*

The significance of this modification is that it directly shows the importance of initiative in a population if space is limited. For example, in a 96% saturated world, many agents do not get the opportunity to move because spaces that are moved away from are spaces of low happiness for a certain color. In this world, high-happiness spaces are at a premium and once they are taken, no more integration or segregation can occur. In contrast, in a world with many empty spaces, segregation is relatively low, because isolated agents can be happy and small groups of agents can be happy because they do not share a border with a group of differently colored agents. The underlying suggestion that this modification makes is that with high population density, a high level of segregation is less likely to occur. If agents are too picky, or not picky enough, very low levels of segregation can be seen. With mid-level population density, very selective agents can achieve high levels of segregation up to a certain critical point. Distance does not play as critical a role as expected in this test, but has more profound effects in test 1B. The reason distance is not an important factor in this simulation is because there is no detriment to moving a long distance. If two cells have an acceptable happiness, an agent will choose the cell that is closer, but if only one acceptable cell exists, the agent will move to that cell, regardless of how far it is.

## V. Modification 1B (Distance and Happiness Improvement Bias)

### A. Behavior and Characteristics

Modification 1B is similar to 1A in that an agent will attempt to move to the closest location possible that will fulfill its happiness quota, as designated by the initial  $p$  value. If a location cannot be found that will fulfill this quota, the agent will move to the closest location that will improve its current happiness. This small addition to 1A promotes the population to segregate in a way that is more similar to the original, unaltered Schelling model than to modification 1A itself.

### B. Experimentation and Graph

Using the selected segregation metric, we ran 63 trials on the original Schelling model provided to us: 1 for 1200, 800, and 400 agents of each type at each happiness threshold level from 0 to 1 at increments of 0.05.

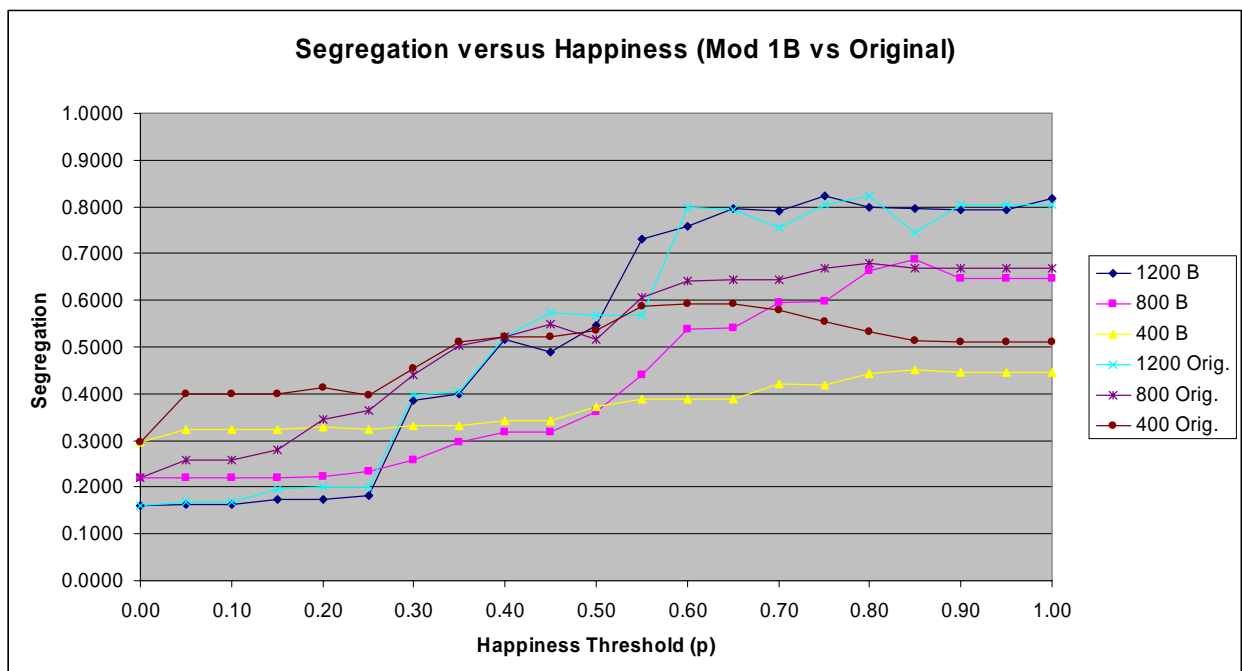


Figure 7

### C. 2400 Agents

Based on the parameters defined for agent movement in this modification, we predicted very similar segregation values to the original Schelling model. The original model has each agent choose a location that betters its happiness randomly, while this modification requires that this location is instead chosen based on distance from the current cell. When the world is 96% filled with agents, the number of empty cells is small enough to negate the difference

between these two versions of the Schelling model. As you will see with a more sparsely populated world, the number of empty cells effects the resemblance to the original model.

#### D. 1600 Agents

The increase in segregation found at the  $p$  value of .25 is due to each agent's preference of surrounding neighbors of the same color increasing from 2 to 3. This is consistent with all other simulations we ran with the same neighborhood parameters. As the  $p$  value increases further, however, we see a slower increase in segregation than the original model. This can be attributed to agents moving "slower", or a shorter distance with each step.

#### E. 800 Agents

Similar to the world populated with 1600 agents, the increase in empty cells causes the difference between random swapping and distance based swapping to become more apparent. In this simulation, 68% of the world is empty, so the change between the original and this modification is much greater. As the  $p$  value approaches 1, however, the number of available cells that fulfill the agents' happiness requirements decrease, which is why the lines labeled 400 original and 400 B in **Figure 7** appear to approach each other.

## VI. Analysis of Modification 1A versus 1B

### A. Comparative Attributes and Graph

Both variations on this modification change the movement behavior of the agents so that each agent moves to closer cells as opposed to cells that are further away. The rules for agent movements are as follows:

- If the agent's happiness is  $\geq p$ , the agent will not move.
- If the agent's happiness is  $< p$ , the agent moves to the closest cell such that its happiness will be  $> p$  after moving to the new cell.
- In version A, If no cell fulfills the agent's happiness threshold, it will not move.
- In version B, If no cell fulfills the agent's happiness threshold, it will move to the closest cell that *improves* its current happiness.

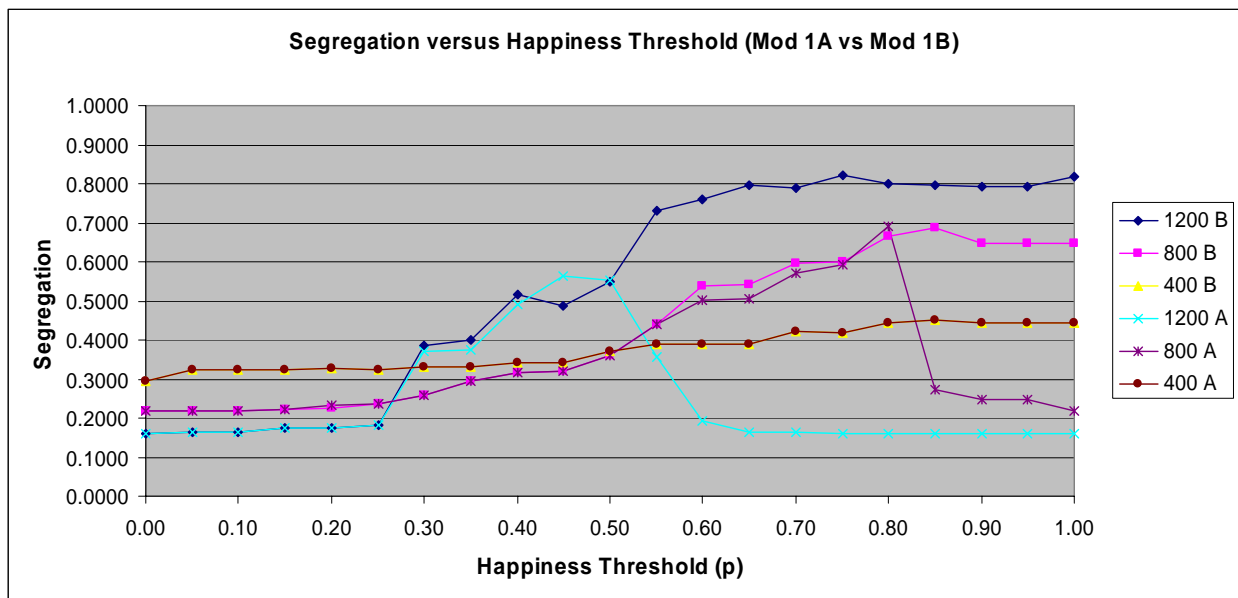


Figure 8

### B. 2400 Agents

When the world is as crowded as it is with 1200 agents of each color, the requirements for movement stated in 1B allow for agents to move into previously occupied cells that used to hold an agent of the same color (as opposed to 1A, when an agent would only leave a cell for a space that immediately fulfilled its happiness quota). This is the reason that the drop off seen in 1A as the  $p$  value exceeds .5 is not seen in 1B.

### C. 1600 Agents

Both modifications 1A and 1B share similar segregation levels when there are 800 agents of each color present in the world. When the  $p$  value reaches .8 however, modification 1A drops off steeply. As stated previously, the change in 1B that allows an agent to move *even if*

its happiness value is not met causes agents to continue to move, even after all the “best” cells have been filled.

#### D. 800 Agents

Since there is such a low density of agents when only 400 of each race are present, a happiness of 1 for each agent can usually be achieved in only 1 move. Since agents reach their full happiness potential so quickly, there is not enough time to segregate or integrate, as is clearly evidenced in the data. Both modifications 1A and 1B show a degree of segregation that remains almost linear when tracked from the initial  $p$  value of 0 up to 1.

## VII. Modification 2 (Restricted Neighborhood Calculation)

### A. *Behavior and Characteristics*

After some discussion of variation possibilities, we began to contemplate why agents located at the four diagonals of an individual agent should be important in determining its happiness. We decided that, in a real world situation, one would likely be far more concerned with those people directly adjacent to him or his property than those on the corners. In other words, those agents that are actually touching an individual agent ought to be the ones that determine its happiness.

From a social perspective, we came to the conclusion that segregation of different types of people was inevitable. However, we decided that if everyone's concept of personal space was changed slightly, that the progress of segregation as a whole could either be slowed or even asymptotically halted. More explicitly, if every single person in a certain town or city based their happiness on the similarity of four neighbors, instead of eight, that it would be more difficult to wholly segregate that town.

We decided to put our idea of "what happiness really should be" into practice by modifying the "sight" of each agent such that it would only consider the agents to the top, bottom, left, and right of it in determining its happiness. We predicted that the simulations would probably take longer to terminate, and create smaller clumps of agents than the original Schelling model.

### B. *Experimentation and Graph*

We decided to run 63 tests of our modification: one for 1200, 800, and 400 agents of each type at each happiness threshold level from 0 to 1 in increments of 0.05

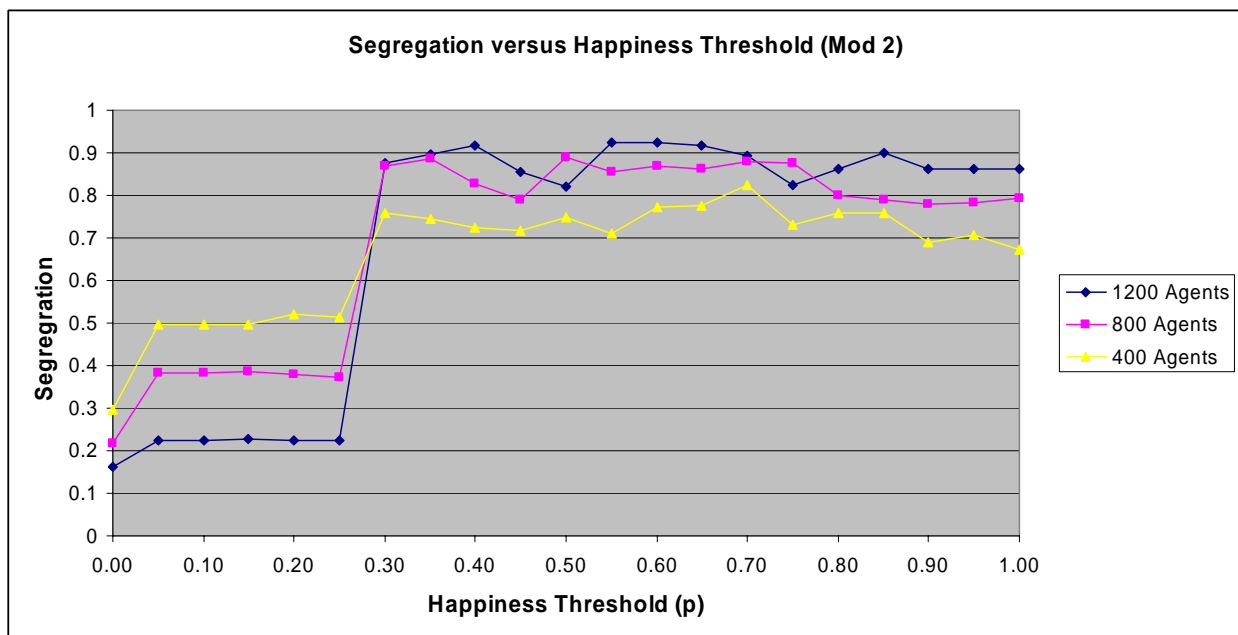


Figure 9



### C. Behavior Analysis

The results of our trials were certainly unexpected. The first trials (those that tested the agents with a happiness threshold of 0 to 0.25) made us feel quite smug in our apparent accomplishment. We had desegregated the masses! The simulations all terminated quite quickly, and usually without much segregation as measured by our metric. Testing the agents at a threshold of 0.3, however, changed everything.

The entire population began what could only be described as an unending dance. Pairs, trios, and quartets of blues and reds formed within the blink of an eye. Eventually, the groups would “dance” their way over to other clusters, creating more segregation. The most fascinating difference between this variation and the original Schelling model was not the speed with which the segregation occurred (which was slower, as predicted), but the fact that the simulation never seemed to reach a state of equilibrium. There were always agents fluctuating between groups, and groups changing shape and size as a result.

The trials after the increase of the segregation index found at the  $p$  value of 0.3 followed with practically the same results. The only other notable increases in segregation occurred at the thresholds of 0.55 and 0.8. We eventually came to the conclusion that since we had chosen to limit each agent’s happiness calculation to 4 surrounding agents (top, bottom, left, and right), that it was only logical that there ought to be three major shifts in segregation. The first, at 0.3, was most major because this was the first time that any agent needed more than one other agent of its own color adjacent to it in order to fully satisfy its happiness requirements. Thus, segregation skyrocketed, as many more agents of the same color grouped together for their own collective good. It follows then, that increases at 0.6 and 0.8 are justified by the idea that these are the first times that any individual agent needs more than 2 or 3 agents of like type to satisfy its desire for happiness.

The fact that the agents were in constant flux after a threshold of 0.25 was more difficult to explain, but we eventually recognized that our “sight” metaphor for our variation was more meaningful than we originally thought. Since each agent can only satisfy its happiness requirements in four ways (an agent above, below, or to the left or right), its “sight” is effectively degraded. In other words, the potential for an agent to have its desire for happiness satisfied in a group is significantly lessened if it is open on any one of its sides. Since an agent in the original Schelling model could get its happiness “fix” in 8 different ways, it was actually far *more* likely to be content with its position. This means that an agent in our variation is more likely to need to move, which creates an oscillation, as moving agents cause other agents in need of happiness to move with them.

### D. Analysis and Summary

From a social perspective, one could say that our variation on the Schelling model actually only decreased the intelligence of the agents instead of offering some sort of boon to their society. The agents were almost as likely to segregate; they simply took longer to do it. The loss of 4 possible ways of achieving happiness even made the agents in our simulation *look* idiotic; as their movements were often rather lemming-like (a group would sometimes simply translate from one side of the map to the other as one agent followed another which followed another which followed another...).

## VIII. Conclusion

### A. *Real-World Parallels for Modification 1A and 1B*

The reason modification 1A is significant and interesting is because it models the behavior of individuals who prefer moving shorter distances. If an individual is given multiple options, each of which fulfills its needs, it is often true that the individual will choose the simplest option. For the most part, even if the options have differing degrees of happiness, and if each option fulfills the individual's needs, the easiest option will be taken. In addition to the bias towards short distance movement, agents also exhibit an aversion to any movement if the end result is not a fulfillment of their happiness threshold.

Modification 1B models a more pragmatic temperament that many individuals have. While 1A is an "all-or-nothing" approach in that no movement is made if the happiness threshold is not met, the 1B modification is a piecemeal approach in that an agent will move to the closest cell that increases its happiness. This causes a slower shift in populations but generally leads to happier agents.

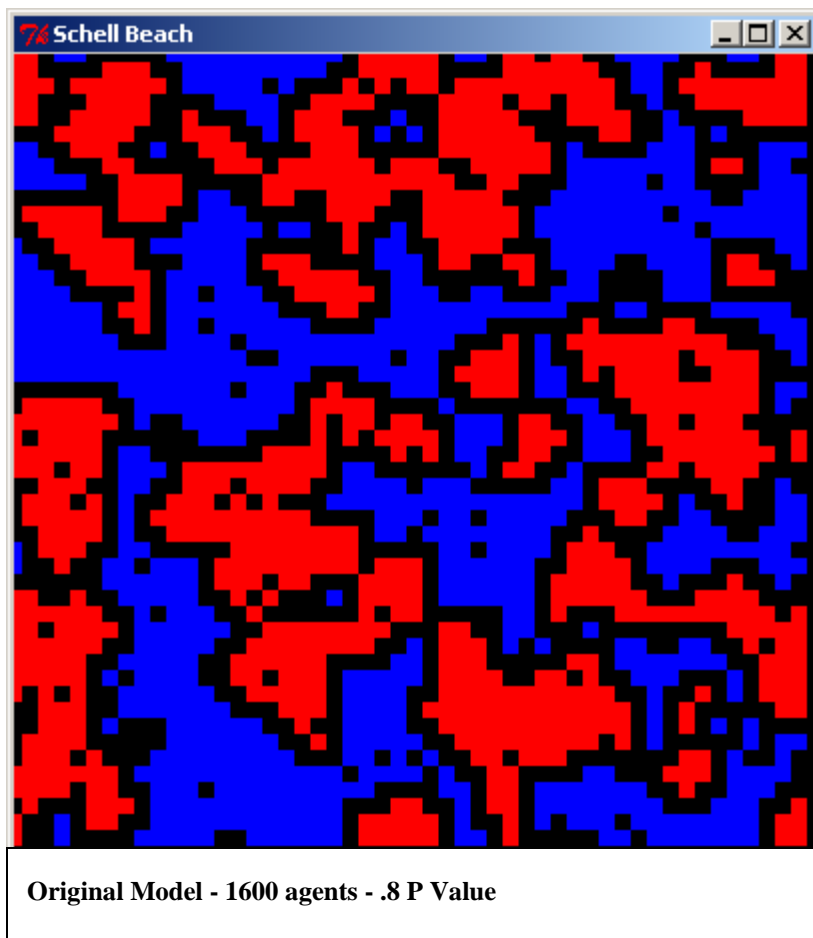
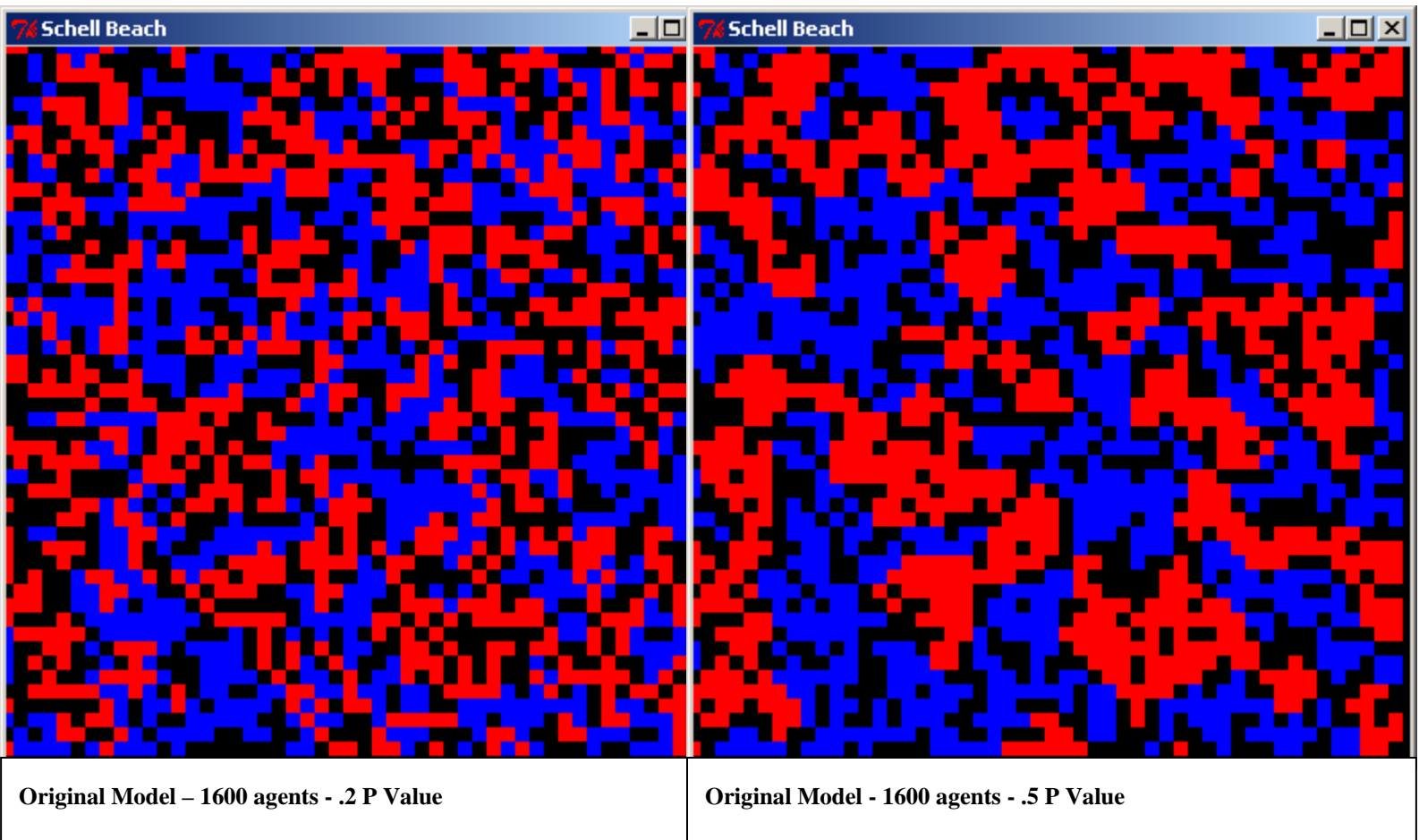
### B. *Real-World Parallels for Modification 2*

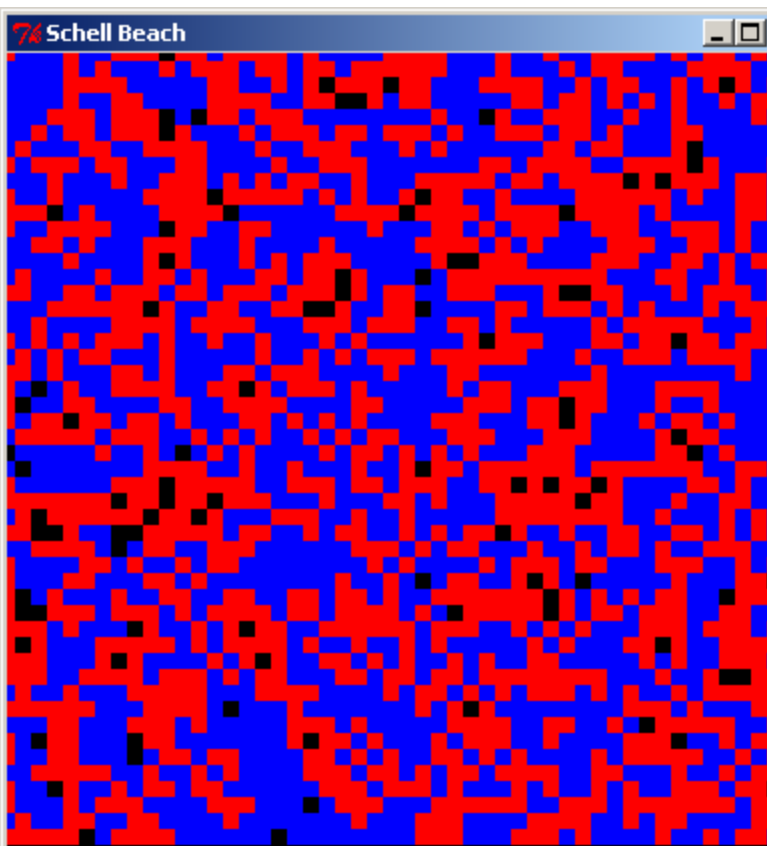
The reason modification 2 is significant is because many individuals disregard the outskirts of their neighborhood when determining how happy they are. What matters most to these individuals are close friends and individuals they come in contact with on a daily basis. Limiting the neighborhood calculations to individuals on the four main sides of an individual leads to more movement in agents because friend bases are smaller. Additionally, this modification facilitates formations of chains of movements because agents that are split up from their small groups are unhappy and seek new places of happiness.

### C. *Impacts of the Schelling Model*

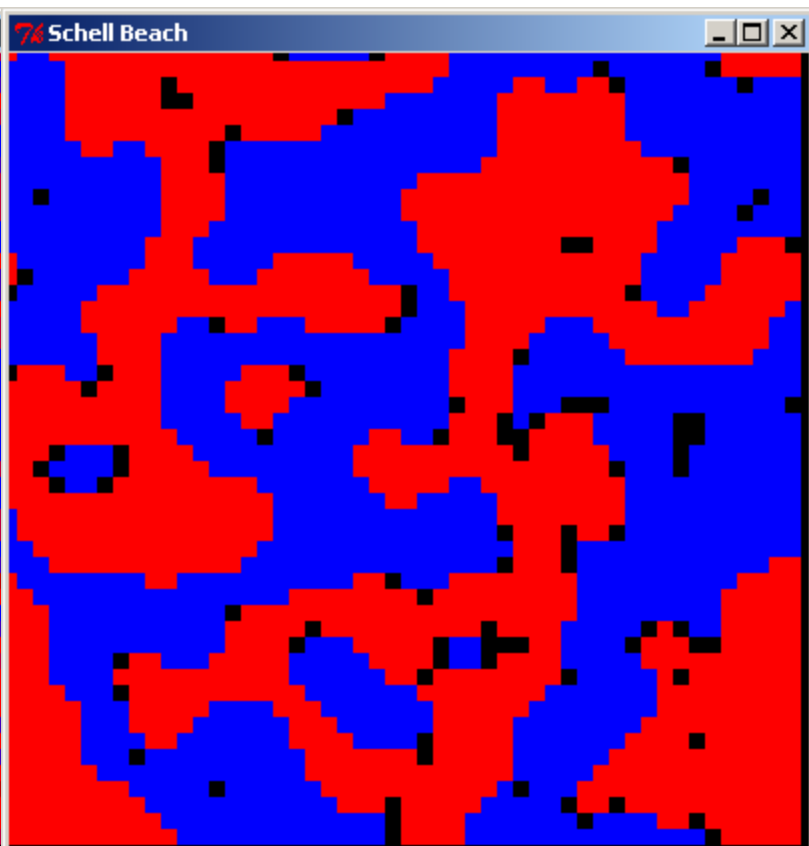
The Schelling Segregation Model has been used for many years to study the segregation of populations. A major factor that must always be remembered is that the behavior of individual agents has major impacts on the behavior of the population as a whole. In our experimentation, we found that changing the agents so that they have a distance bias is an effective way to reduce large distance movements by agents, more realistically mimicking human nature. Though many of the movements remained the same, large shifts in the population were much less apparent in tests where this modification was used. Modification 2 also serves to make the model more realistic because agents are less inclined to view their neighborhood as a whole and cliques start to form.

We can learn many things from the modifications we made to the Schelling model. First, we noticed the power a distance bias had on overpopulated areas where space is at a premium. Second, we were able to deduce that a short distance bias causes smaller population shifts. Third, we realized that smaller neighborhoods cause more cliques to form and lead to more segregation over time. All of these deductions give us insight into the societal workings of population movement and allow use to detect patterns in social behavior.

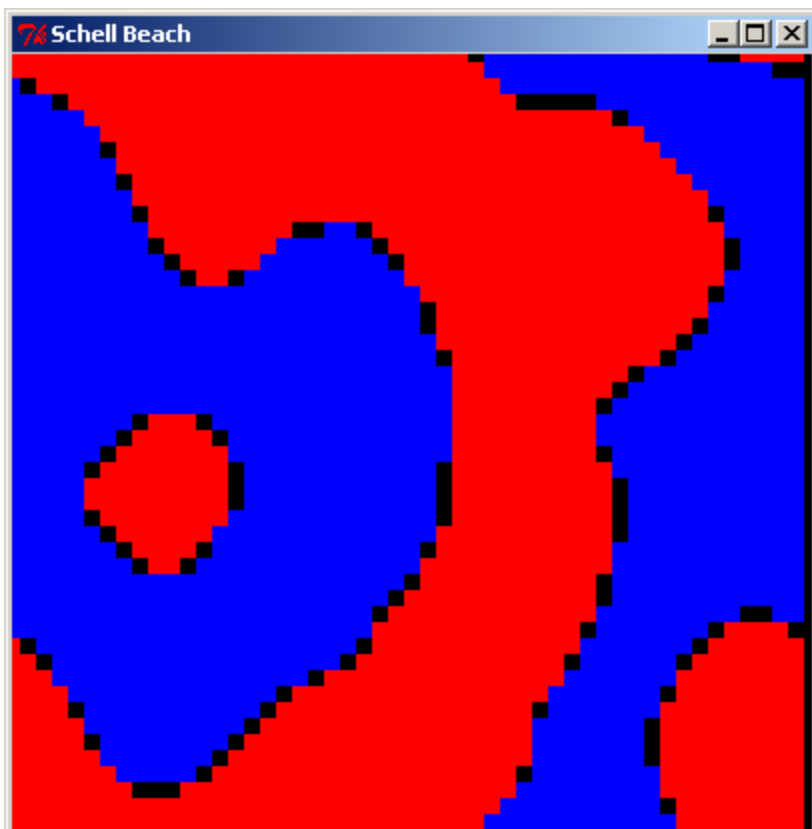




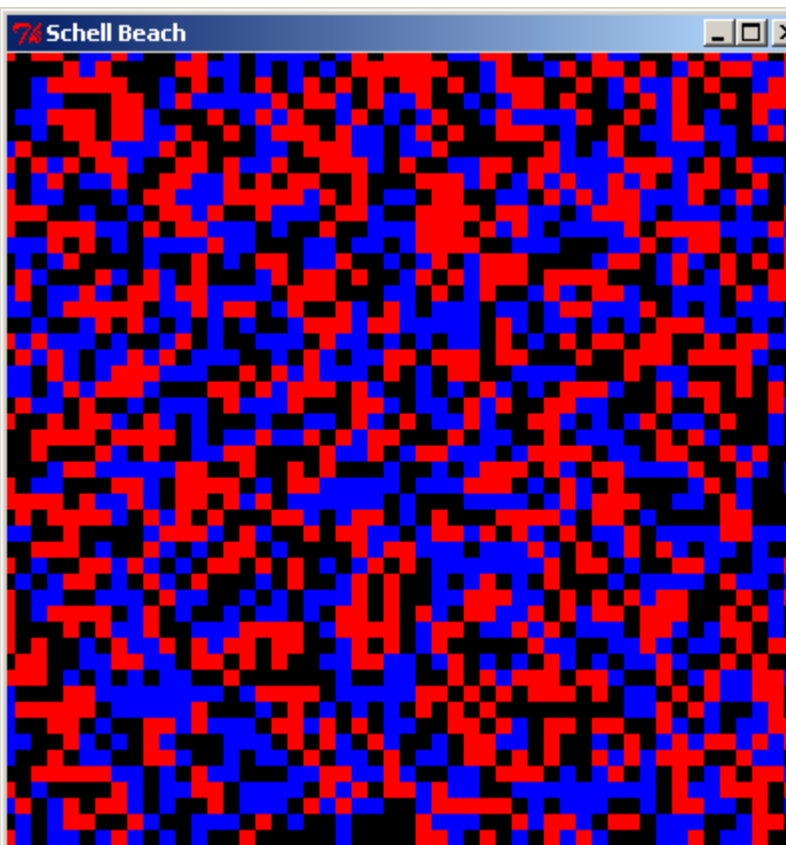
Original Model – 2400 agents - .2 P Value



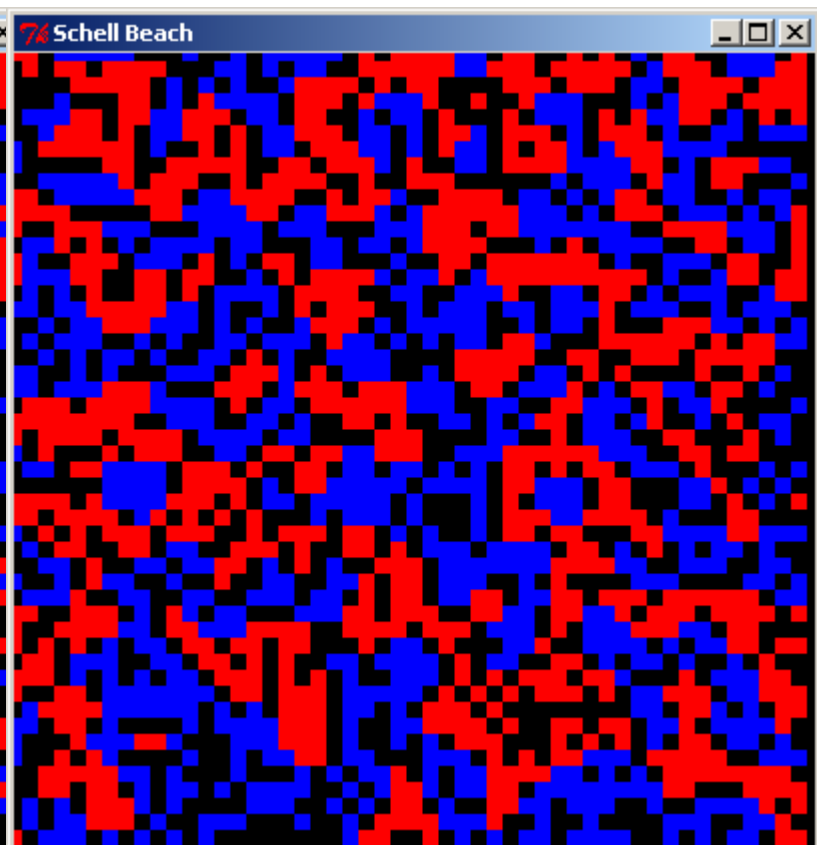
Original Model - 2400 agents - .5 P Value



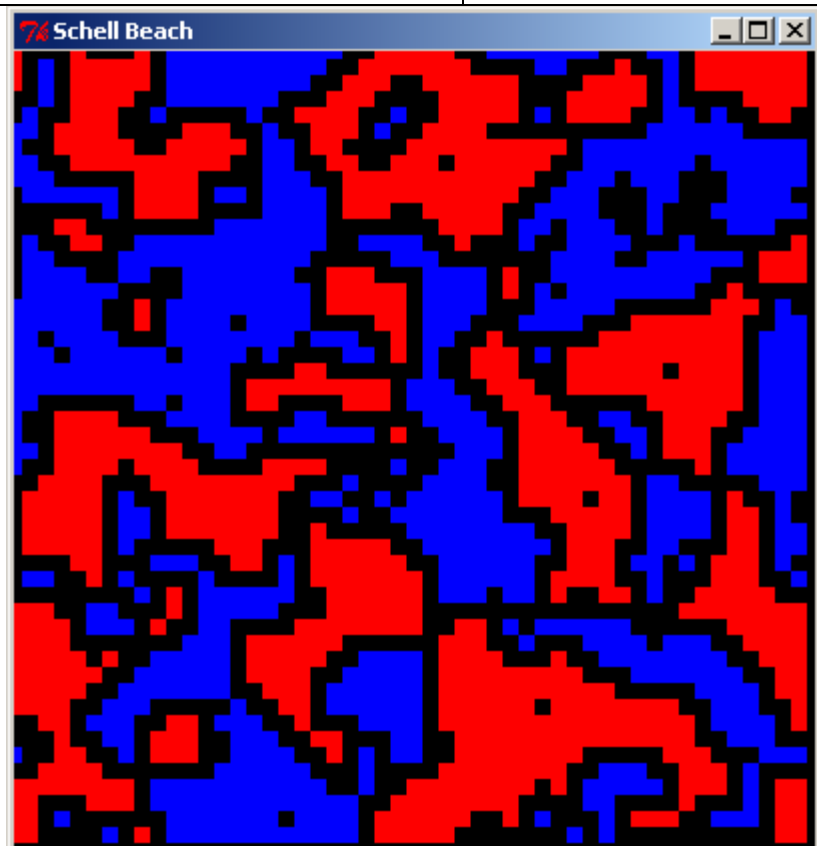
Original Model - 2400 agents - .8 P Value



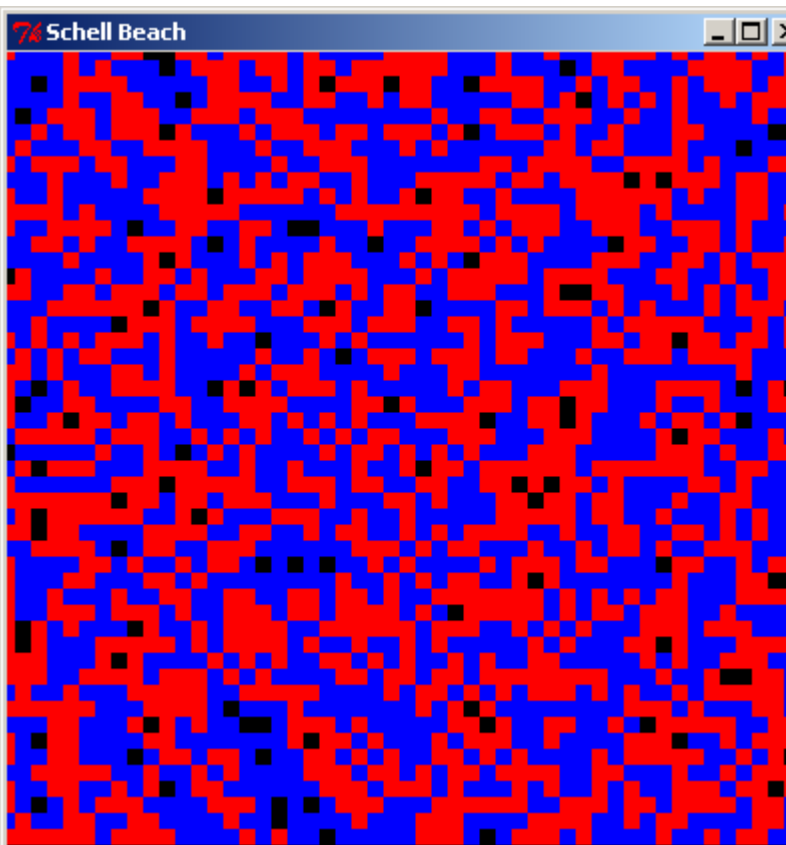
Modification 1A – 1600 agents - .2 P Value



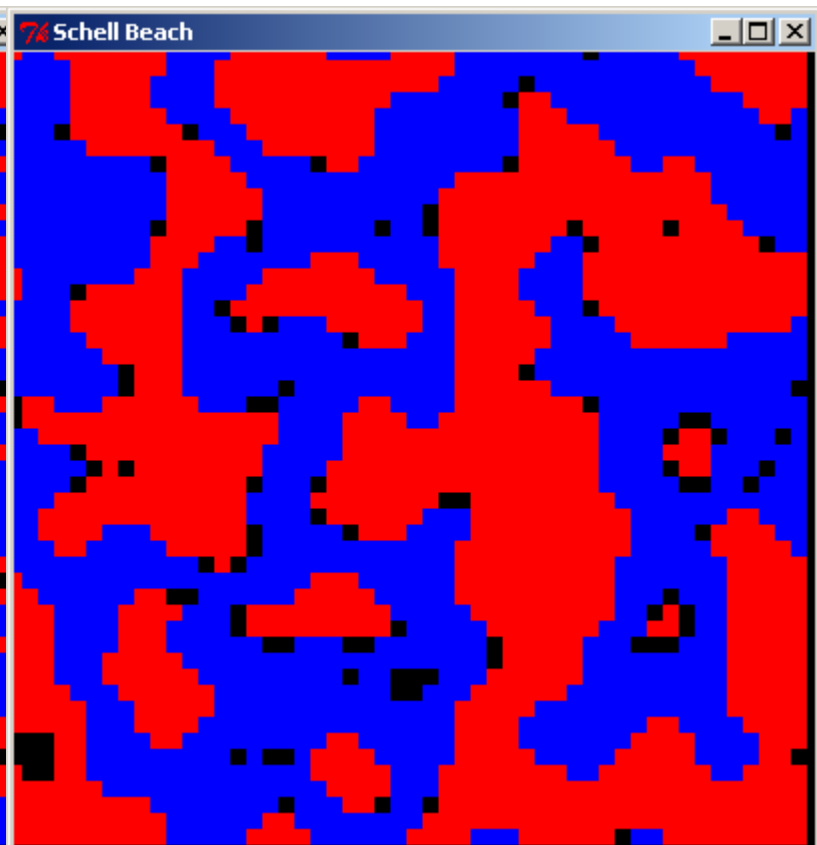
Modification 1A - 1600 agents - .5 P Value



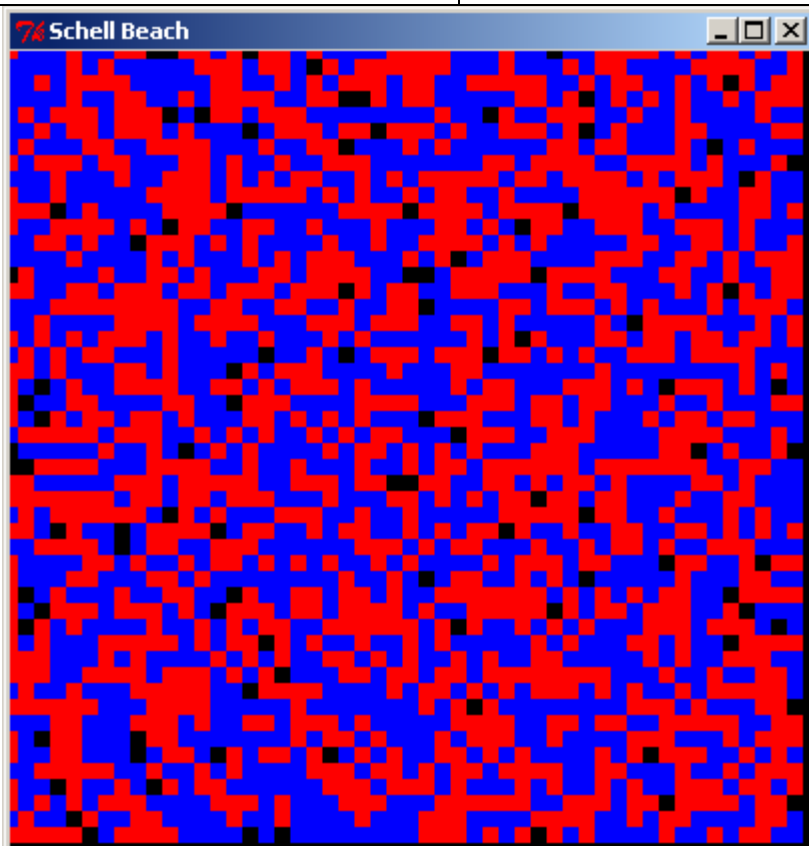
Modification 1A - 1600 agents - .8 P Value



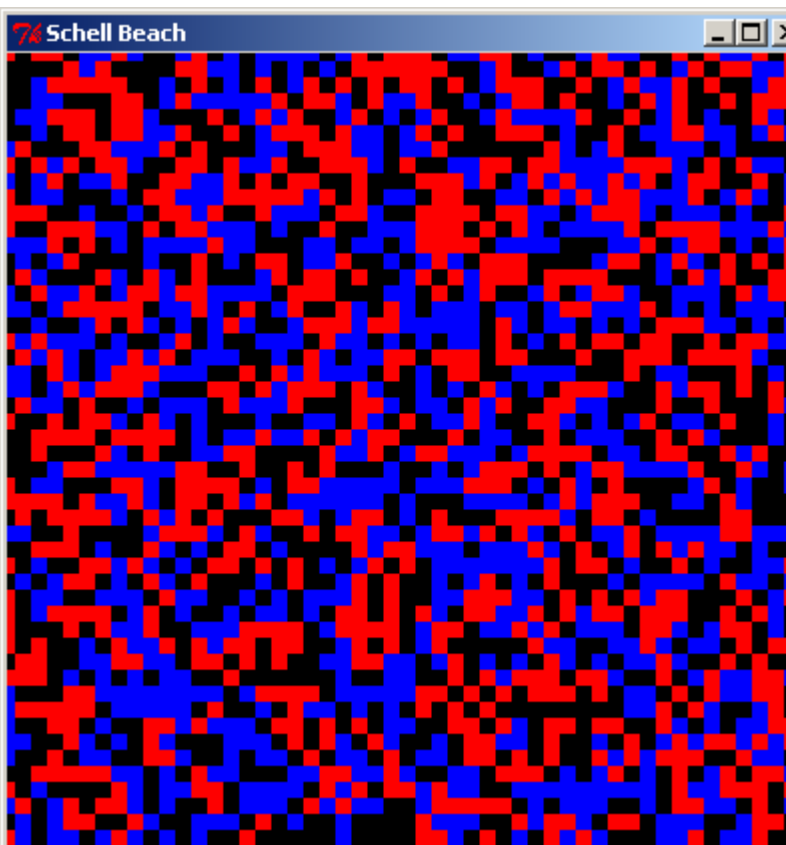
Modification 1A – 2400 agents - .2 P Value



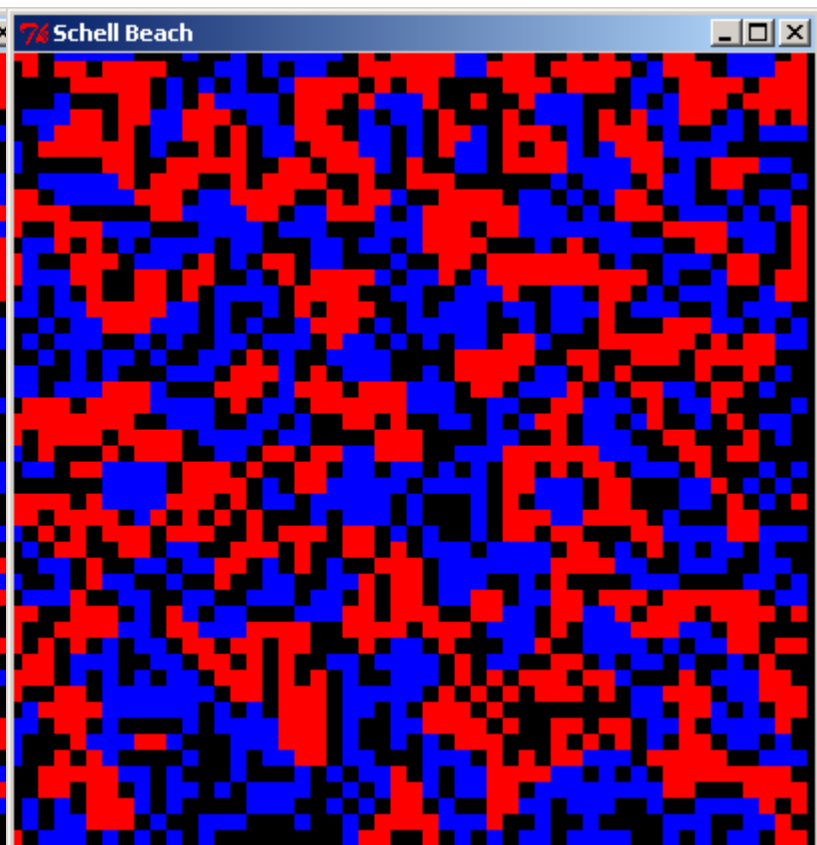
Modification 1A - 2400 agents - .5 P Value



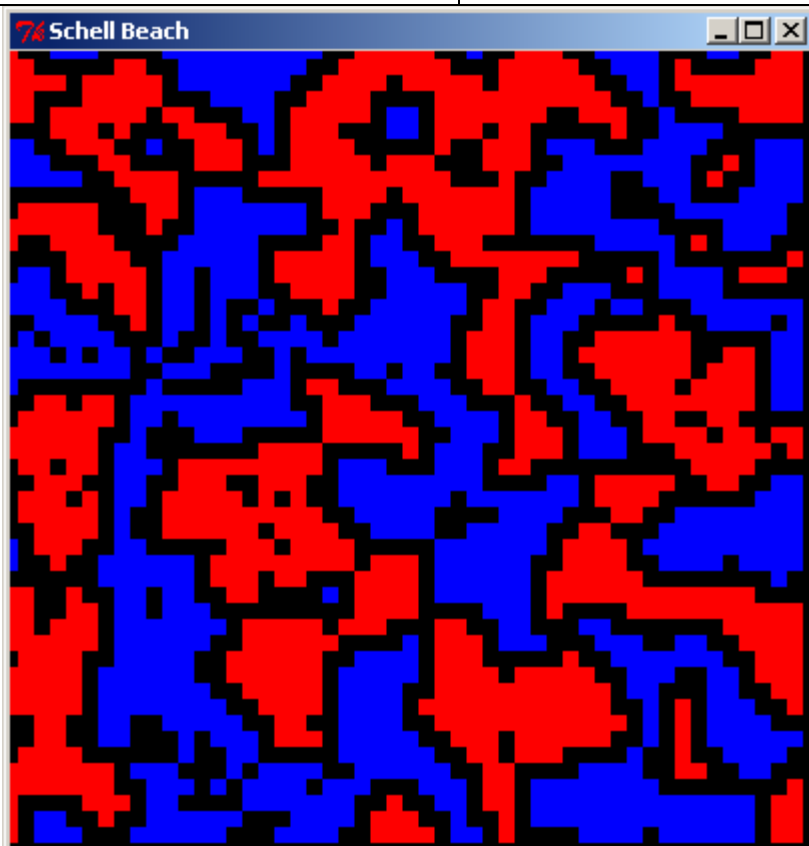
Modification 1A - 2400 agents - .8 P Value



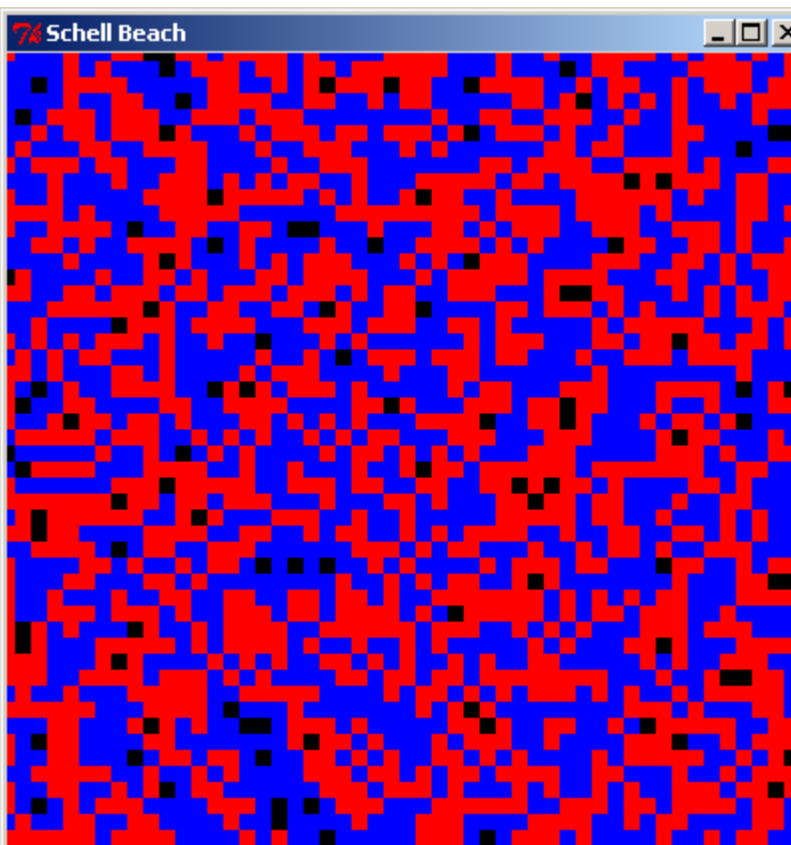
Modification 1B – 1600 agents - .2 P Value



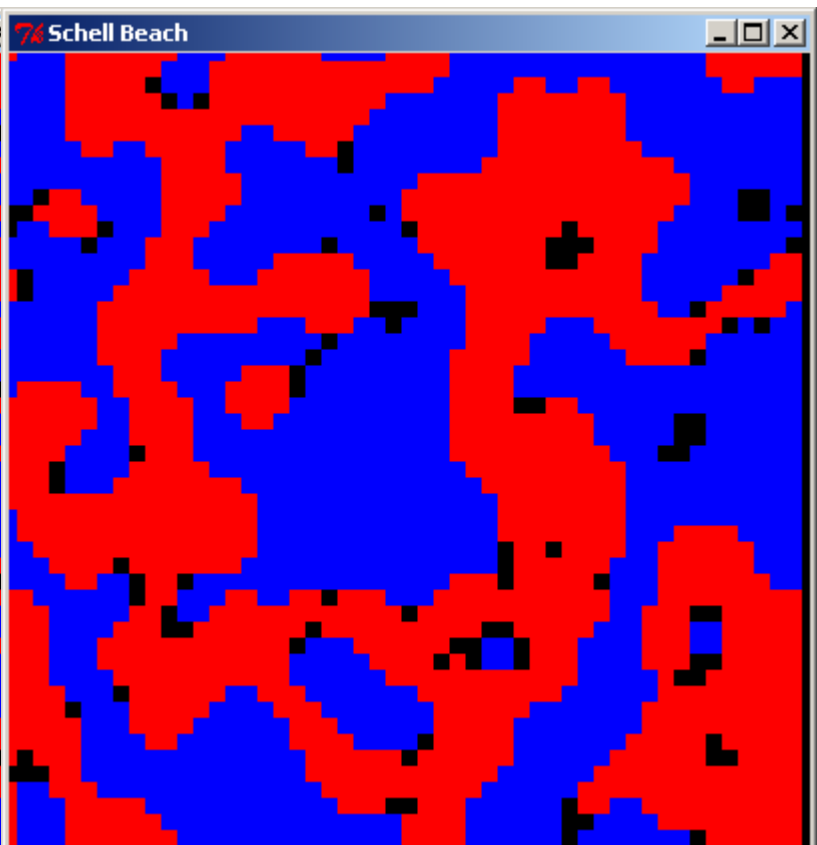
Modification 1B - 1600 agents - .5 P Value



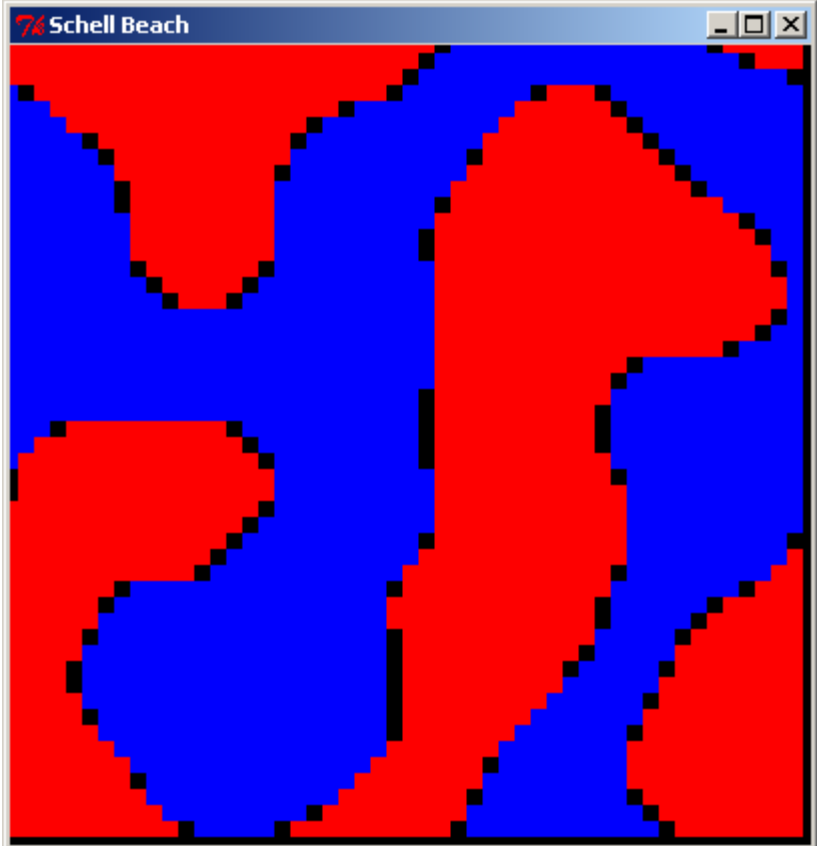
Modification 1B - 1600 agents - .8 P Value



Modification 1B – 2400 agents - .2 P Value

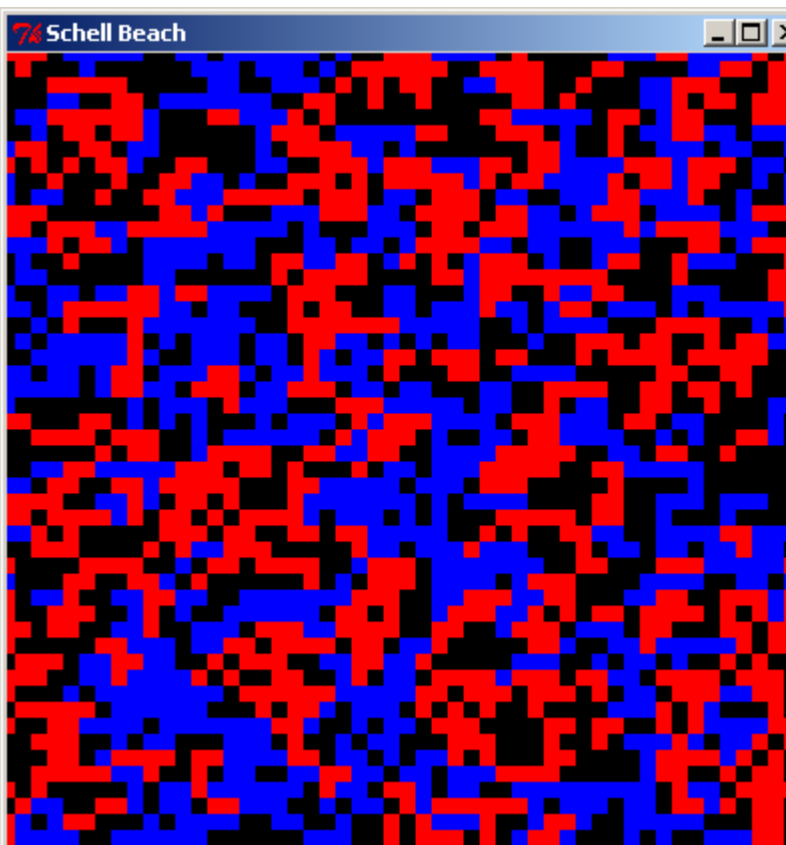


Modification 1B - 2400 agents - .5 P Value

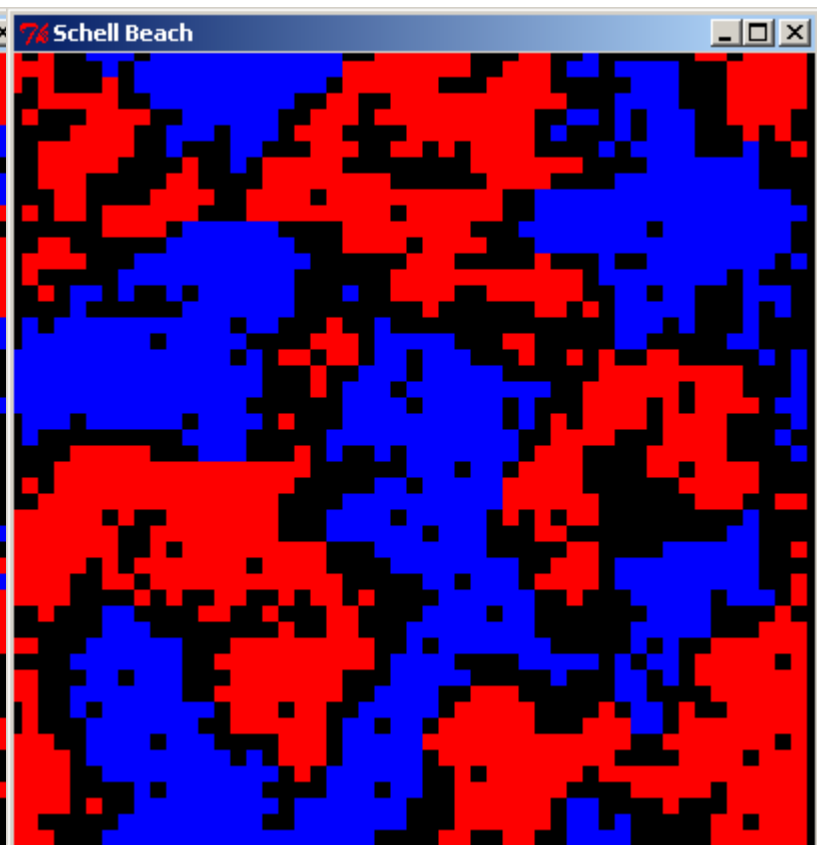


Modification 1B - 2400 agents - .8 P Value

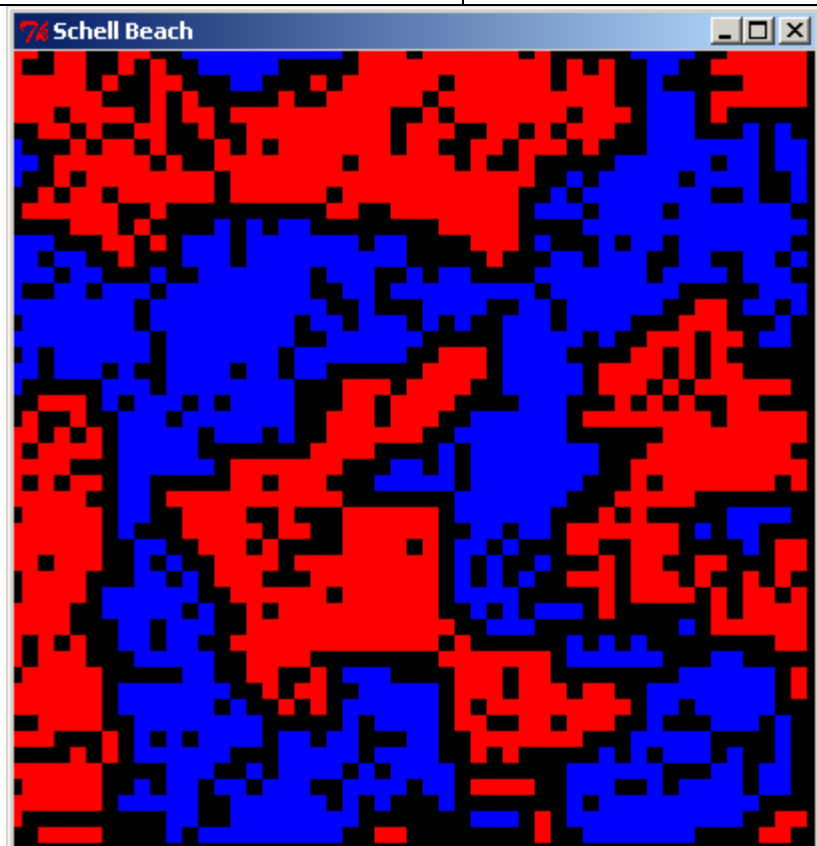




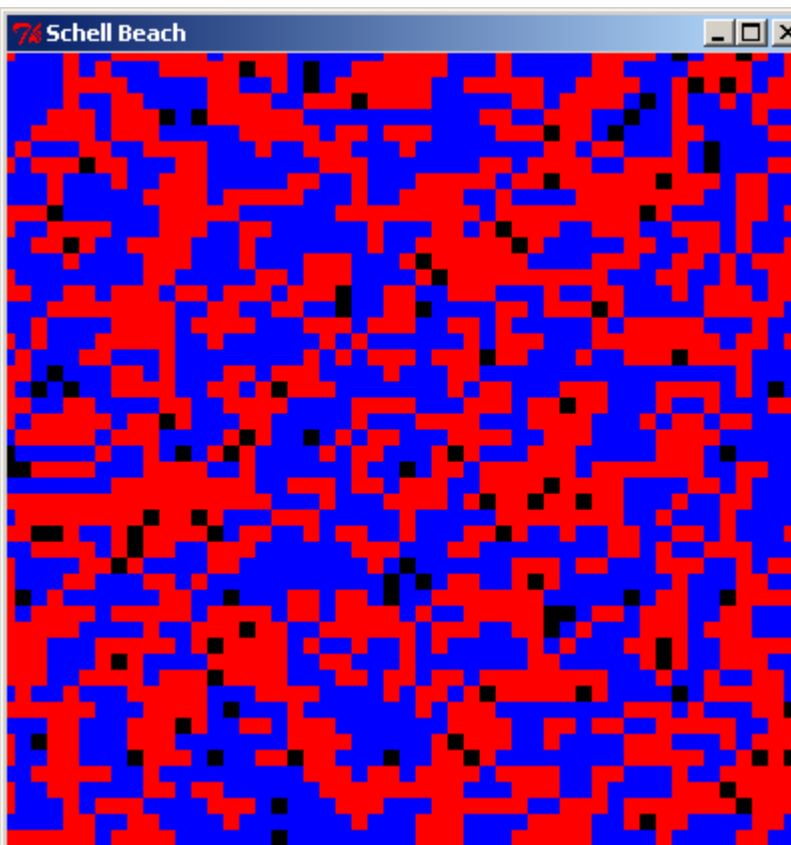
Modification 2 – 1600 agents - .2 P Value



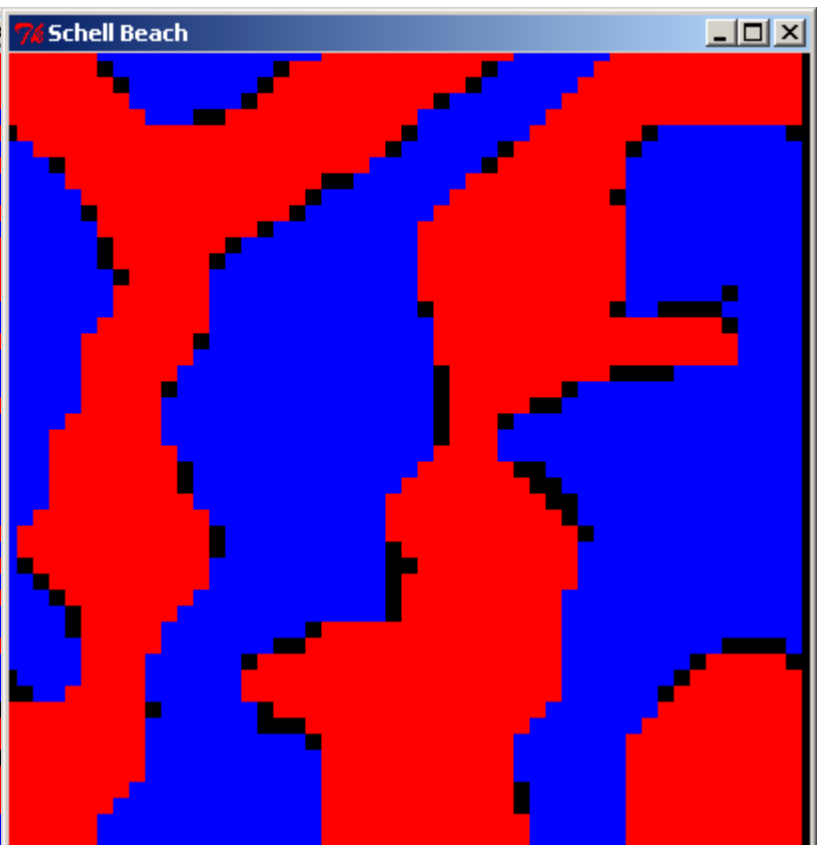
Modification 2 - 1600 agents - .5 P Value



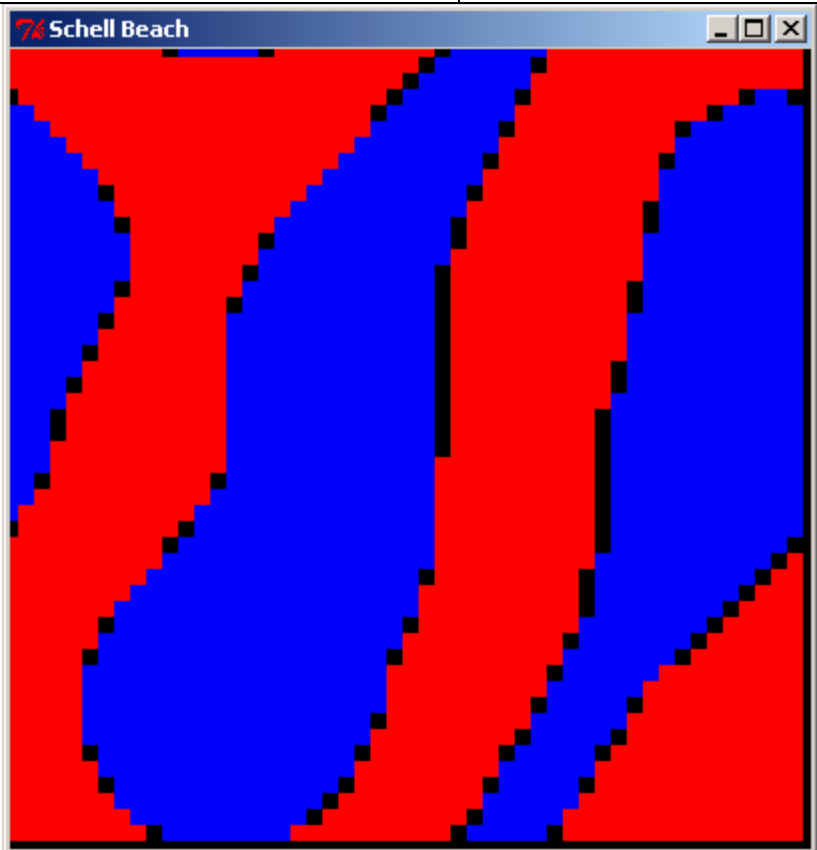
Modification 2 - 1600 agents - .8 P Value



Modification 2 – 2400 agents - .2 P Value



Modification 2 - 2400 agents - .5 P Value



Modification 2 - 2400 agents - .8 P Value

Original Model P Value	Segregation Values for Varying Agent Numbers		
	2400	1600	800
0.00	0.1608	0.2188	0.2950
0.05	0.1683	0.2575	0.4000
0.10	0.1683	0.2575	0.4000
0.15	0.1967	0.2788	0.4000
0.20	0.2008	0.3463	0.4125
0.25	0.2008	0.3638	0.3975
0.30	0.3958	0.4400	0.4550
0.35	0.4050	0.5038	0.5100
0.40	0.5225	0.5225	0.5225
0.45	0.5725	0.5486	0.5225
0.50	0.5667	0.5163	0.5350
0.55	0.5667	0.6050	0.5875
0.60	0.7983	0.6400	0.5925
0.65	0.7925	0.6450	0.5925
0.70	0.7550	0.6438	0.5775
0.75	0.8033	0.6675	0.5550
0.80	0.8225	0.6788	0.5325
0.85	0.7450	0.6675	0.5125
0.90	0.8033	0.6675	0.5100
0.95	0.8033	0.6675	0.5100
1.00	0.8033	0.6675	0.5100

Mod 1A P Value	Segregation Values for Varying Agent Numbers		
	2400	1600	800
0.00	0.1608	0.2188	0.2950
0.05	0.1683	0.2575	0.4000
0.10	0.1683	0.2575	0.4000
0.15	0.1967	0.2788	0.4000
0.20	0.2008	0.3463	0.4125
0.25	0.2008	0.3638	0.3975
0.30	0.3958	0.4400	0.4550
0.35	0.4050	0.5038	0.5100
0.40	0.5225	0.5225	0.5225
0.45	0.5725	0.5486	0.5225
0.50	0.5667	0.5163	0.5350
0.55	0.5667	0.6050	0.5875
0.60	0.7983	0.6400	0.5925
0.65	0.7925	0.6450	0.5925
0.70	0.7550	0.6438	0.5775
0.75	0.8033	0.6675	0.5550
0.80	0.8225	0.6788	0.5325
0.85	0.7450	0.6675	0.5125
0.90	0.8033	0.6675	0.5100
0.95	0.8033	0.6675	0.5100
1.00	0.8033	0.6675	0.5100

<b>Mod 1B</b> <b>P Value</b>	<b>Segregation Values for Varying Agent Numbers</b>		
	<b>2400</b>	<b>1600</b>	<b>800</b>
0.00	0.1608	0.2188	0.2950
0.05	0.1633	0.2200	0.3225
0.10	0.1633	0.2200	0.3225
0.15	0.1730	0.2213	0.3225
0.20	0.1738	0.2238	0.3275
0.25	0.1825	0.2350	0.3225
0.30	0.3858	0.2588	0.3325
0.35	0.4008	0.2956	0.3325
0.40	0.5158	0.3174	0.3425
0.45	0.4883	0.3188	0.3425
0.50	0.5475	0.3613	0.3725
0.55	0.7308	0.4413	0.3886
0.60	0.7583	0.5375	0.3888
0.65	0.7967	0.5413	0.3886
0.70	0.7908	0.5950	0.4225
0.75	0.8233	0.5988	0.4175
0.80	0.7992	0.6638	0.4425
0.85	0.7958	0.6863	0.4500
0.90	0.7942	0.6475	0.4450
0.95	0.7942	0.6475	0.4450
1.00	0.8192	0.6475	0.4450

<b>Mod 2</b> <b>P Value</b>	<b>Segregation Values for Varying Agent Numbers</b>		
	<b>2400</b>	<b>1600</b>	<b>800</b>
0.00	0.1608	0.2188	0.2950
0.05	0.2233	0.3838	0.4950
0.10	0.2233	0.3838	0.4950
0.15	0.2267	0.3863	0.4950
0.20	0.2225	0.3788	0.5200
0.25	0.2250	0.3725	0.5150
0.30	0.8767	0.8675	0.7575
0.35	0.8975	0.8850	0.7450
0.40	0.9167	0.8288	0.7250
0.45	0.8567	0.7900	0.7175
0.50	0.8217	0.8913	0.7475
0.55	0.9258	0.8563	0.7100
0.60	0.9242	0.8688	0.7725
0.65	0.9167	0.8638	0.7750
0.70	0.8917	0.8800	0.8225
0.75	0.8242	0.8775	0.7325
0.80	0.8625	0.7988	0.7595
0.85	0.8992	0.7900	0.7600
0.90	0.8625	0.7788	0.6900
0.95	0.8617	0.7813	0.7084
1.00	0.8633	0.7925	0.6725

